INTERACTIVE FUZZY LINEAR PROGRAMMING

-- AN EXPERT LP SYSTEM

by

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CHAPTER 1 INTRODUCTION

At the turn of the century, the Swiss historian Jakob Burkhardt, who, unlike most historians, was fond of guessing the future, once confided to his friend Friedrich Nietzsche the prediction that the twentieth century would be "the age of oversimplification" [19].

In the early twentieth century, reducing complex real-world systems into precise mathematical models became the main trend in science and engineering. In the middle of this century, Operations Research (OR) was started to apply to real-world decision making problems. Since that time, OR has become one of the most important fields in science and engineering. However, the real-world situations are very often not crisp and deterministic and thus cannot be described precisely. This imprecise nature is actually fuzziness rather than randomness. Therefore, the traditional OR approaches may not really be suitable in solving real-world decision making problems.[132]

The role of fuzzy sets in decision processes is best described in the original statements of Bellman and Zadeh [1], That is:

> Much of the decision-making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely. To deal quantitatively with imprecision, we usually employ the concepts and techniques of probability theory and, more particularly, the tools provided by decision theory, control theory and information theory. In so doing, we are tacitly accepting the premise that imprecision -- whatever its nature

-- can be equated with randomness. This, in our view, is a questionable assumption.

Bellman and Zadeh gave the major nature of fuzziness which is different from randomness. Fuzziness is a type of imprecision which is associated with fuzzy sets, that is, classes in which there is no sharp transition from membership to nonmembership. For example, the class of "beautiful girls," "intelligent men," "creditworthy customers," and so on, where objects -- girls, men and customers -- are characterized by such commonly used adjectives as "beautiful", "intelligent", "creditworthy", etc., are fuzzy sets. Bellman and Zadeh indicated that, in sharp contrast to the notation of a class or a set in mathematics, most of the classes in the real world do not have crisp boundaries which separate those objects which belong to a class from those which do not. This is especially so in areas in which human judgment, evalution, and decisions are important, such as decision making, reasoning, learning, and so on.[1][132]

Zadeh [132] explicitly indicated that, the theory of fuzzy sets is, like its name, a theory of graded concepts -- a theory in which everything is a matter of degree. Therefore, the figures and numeric tables are considered paramount in the study of fuzzy sets theory. This is an important concept in the new generation of operations research. Contemporarily, the decision makers find that it is often confusing to be confronted with OR slang or to be

unable to understand OR publications because they lack the necessary background [136]. Therefore, communication between experts (or analysts) and the decision makers becomes more and more difficult. However, it is not necessary to ask the decision makers to learn a great variety of OR concepts, techniques and mathematics in order to use them. For example, the decision maker does not need to understand the construction, the engine systems and force theory of a car in order to drive the car. Likewise a secretary without any knowledge about computer programs, operation systems, hardware construction, etc., can easily use a word processor. Likewise in OR, communication with the decision makers, which has been neglected in the early years, should be improved. Figures and numeric tables, unlike mathematical slang, difficult functions, etc., provide the best ways to communicate with outsiders (decision makers) who are not good at OR, but who need to use it.

Fuzzy sets theory as previously described is an important field in solving real-world decision making problems. It has been applied to many OR techniques such as linear programming, dynamic programming, nonlinear programming, queuing theory and so on. Among these, linear programming (LP) is considered the most important technique and will therefore be emphasized in this study.

In the decision processes, the decision maker himself is always playing the most important role. Therefore, an

interactive concept is considered a good approach in developing a user-dependent fuzzy LP technique, and is investigated in this study. At the same time, we also want to be able to solve a variety of LP problems. Thus a problem-oriented concept is also considered. The importance of the problem-oriented concept was depicted in the original statement of Simon [81]:

> Perhaps what is most important in making rapid progress towards our goals is that we adopt a problem-oriented point of view. We must let the problem that we are trying to solve determine the methods we apply to it, instead of letting our techniques determine what problems we must be willing and able to tackle. That means we must be willing to apply our entire armory of techniques, both heuristic and algorithmic."

The interactive fuzzy linear programming, in this study, is a symmetric synthesis of Zimmermann's, Werners's, Chanas's and Verdegay's approaches and further provides an expert LP system for solving a specific domain of real-world LP problems.

This study is divided into four chapters. Chapter 1 is an introduction chapter. Chapter 2 provides the overview of the current literature of the fuzzy linear programming approaches. At the same time, A brief introduction of linear programming, fuzzy sets theory and the connection between linear programming and fuzziness are also provided. The main topic of this study, interactive fuzzy linear programming -- an expert LP system is introduced in Chapter 3. In this chapter some advantages and disadvantages of Zimmermann's, Werners's, Chanas's and Verdegay's approaches are discussed.

Five LP problems which can be solved by our interactive fuzzy linear programming are then shown in the second section. An algorithm and flow chart are also presented in this section. In the last section we provide an example in order to more clearly express our interactive fuzzy linear programming approach. Finally, brief concluding remarks and comments on further studies are provided in Chapter 4.

CHAPTER 2 THE LITERATURE SURVEY OF FUZZY LINEAR PROGRAMMING

2.1 Introduction to Linear Programming

While L.V. Kantorovich, a Soviet mathematician and economist, formulated and solved linear programming dealing with the organization and planning of production in 1939[40], G. B. Dantzig, M. Wood and their associates of the U. S. Department of the Air Force developed the general problem of linear programming named by A. W. Tucker[18]. In order to apply mathematical and related techniques to military programming and planning problems, Dantzig proposed "that interrelations between activities of a large organization be viewed as a linear programming type model and the optimizing program determined by minimizing a linear objective function." Conceptually, some concepts originated from Leotief's input-output model. Since that time, linear programming has become an important tool of modern theoretical and applied mathematics. [18][47]

Linear programming is concerned with the efficient allocation of limited resources to activities with the objective of meeting a desired goal such as maximizing profit or minimizing cost. The distinct characteristic of linear programming models is that the interrelations between activities are linear relationships which are the satisfactions of the proportionality and additivity requirements. [4]

The general linear programming problem may be stated symbolically as follows:

$$\begin{array}{ll} \text{maximize} & Z = c^T x \\ \text{subject to} & Ax \leq b \\ & x \geq 0 \end{array} \qquad (2.1)$$

where c is the vector of profit coefficients of the objective function and b is the vector of total resources available. x is the vector of decision variables (or alternatives), and A is the matrix of technical coefficients.

For example, a decision maker may want to design a production schedule with the greatest profit under the current production capacity and current total resources available. He assumes that the available resources such as labor hours (1200), and materials (1550), which are needed to produce the three products A, B and C with the corresponding production amounts x_1, x_2 and x_3, are precisely known. The prices (c = (3, 4, 4)) of the products are also precisely known. At the same time, the technique coefficients (A_1 = (6, 3, 4) and A_2 = (5, 4, 5)) for the current production system are assumed to be constants.

By using the linear programming technique, he formulates the problem as:

$$\begin{array}{llr} \text{maximize} & 3x_1 + 4x_2 + 4x_3 & \text{(profit)} \\ \text{subject to} & 6x_1 + 3x_2 + 4x_3 \le 1200 & \text{(labor hours)} \\ & 5x_1 + 4x_2 + 5x_3 \le 1550 & \text{(material)} \\ & x_1,\ x_2,\ x_3 \ge 0. & \end{array}$$

Naturally, a decision maker may have the following questions:

1. Where are those coefficients from? It is very possible that those input data (coefficients) are actually provided by the decision makers. However, presenting the input data as precise numbers is, sometimes, questionable because of existence of the vagueness.

2. Are the total resources available, 1200 for labor hours and 1550 for material, absolutely not violated in the previous model? The decision maker may find that there exist some alternatives which allow him to have more choices. The possible alternatives may be as follows:

-- He can get extra labor hours from overtime or other labor forces.

-- He can obtain extra material from the supplier by paying more money or searching for other suppliers.

Therefore, the previous precise (crisp) linear programming model is questionable for its ability to provide an optimal decision.

Essentially, the previous model is too much oversimplified to model the real-world complex system, especially when the system involves human aspects. It is

obvious that the imprecise natures are not of randomness but fuzziness.

2.2 Introduction to Fuzzy Sets Theory

The term "fuzzy" was proposed by Zadeh in 1962 [37]. In 1965, Zadeh formally published the famous paper "Fuzzy Sets Theory"[112]. The fuzzy sets theory, was intended to improve the oversimplified model, thereby developing a more robust and flexible model in order to solve real-world complex systems involving human aspects. Furthermore, it helps the decision maker to not only consider the existing alternatives under given constraints (optimize a given system), but to also develop new alternatives (design a system). Thus, the fuzzy sets theory provides a good tool to solve the previous problems.

Actually, fuzzy sets theory has been applied in many fields, such as control systems, optimization theory, artificial intelligence, human behavior, etc. In this study we are concerned with optimization theory, especially concentrating on fuzzy linear programming. Review of OR literature indicates that linear programming is the most important technique for real-world applications, even though there exist a lot of other techniques, such as queuing theory, heuristic approaches, nonlinear programming and so on. Therefore, the purpose of this chapter is to give an overall view of the fuzzy linear programming problems.

2.2.1 Introduction to Fuzzy Sets

In order to be able to clearly distinguish the fuzzy sets from classical (crisp) sets, let us first consider the basic concept of classical sets.

In the classical sets theory, an element may either "belong to" a set A or "not belong to" the set A in the given universe. For example, suppose that there is a rather big target and shooters always hit inside the target (see Figure 2.1). A circle is located in the center of the target. If a shooter hits inside the circle, region A, he is given a title "good shooter". Otherwise, he is called a poor shooter. Shooter a^1 shoots inside the region A, so he is a good shooter. On the other hand, shooter a^2 is a bad shooter. The classical set theory with binary relationship shows some problems. For example, if there are three shooters, a^3, a^4 and a^5, who hit the target within close range of one another (see Figure 2.2), yet only a^3 is within the target, then shooter a^3 is a good shooter and shooters a^4 and a^5 are bad shooters. This is obviously unreasonable.

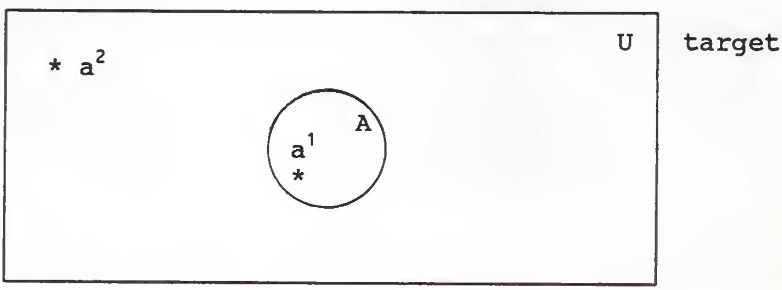


Figure 2.1. The shooter example: a^1 is a good shooter and a^2 is a bad shooter

As a matter of fact, these three people should obtain some similar designation, at least up to a certain degree. Therefore, a measure of the degree to which the shooter belongs to the set "good shooters" should be developed in order to discern how good the shooter is. The set composed

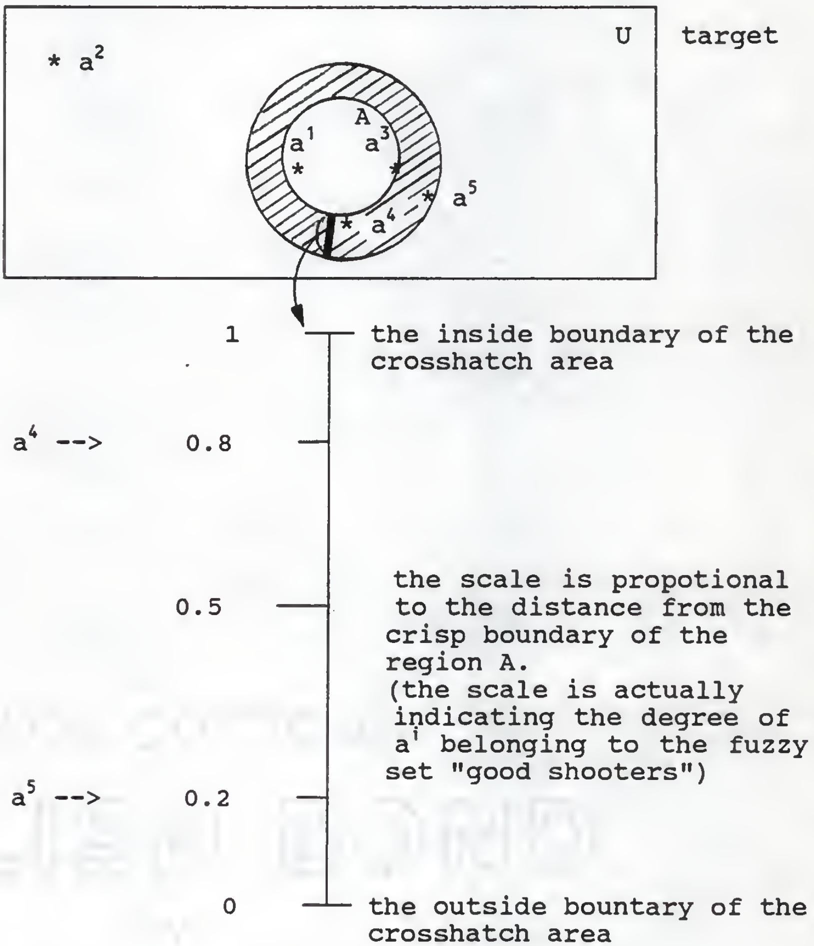


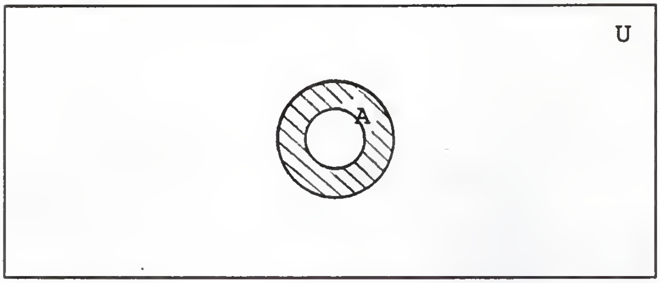
Figure 2.2. The graphic explanation for the basic concept of the fuzzy sets theory.

of good shooters is actually fuzzy because there is no crisp boundary. It is rational to consider the distance from the boundary of the region A as a measure for indicating the degree to which shooter a^i belonging to "good shooters". In Figure 2.2, a^1 and a^3 are absolutely good shooters. On the other hand, a^4 and a^5 are not absolutely good shooters or absolutely bad shooters, but, at least to some degree, good shooters. By giving a numerical measure which is linearly proportional to the distance of each shooter from region A, one can say that shooter a^4 has 0.8 degree of membership in the set of good shooters versus 0.2 for shooter a^5. Of course, the numerical measure can be any number. A normalized measure which is in [0, 1] is always adapted. Therefore, the following table (Table 2.1) lists the grades of membership belonging to the set of good shooters for the five shooters:

Table 2.1. The grades of membership belonging to the set of good shooters

shooter	a^1	a^2	a^3	a^4	a^5
degree of membership	1	0	1	0.8	0.2

The crosshatch area in Figure 2.2 is considered as a vague, cloudy, or fuzzy boundary area (not a crisp, sharp boundary). In addition, one might consider the inside crosshatch boundary area of region A as a fuzzy area (see



target

Figure 2.3. An example of inside fuzzy boundary region

Figure 2.3). In conclusion, the set with a vague, cloudy, or fuzzy boundary is named a fuzzy set.

In order to be able to more clearly understand the fuzzy sets concept, let consider one more example.

Example.

Consider a universe composed of 4 male students with the same height of 5'9", Hans, John, George and Young. That is, U = { Hans, John, George, Young }. The weights for the 4 male students are given as follows:

Hans : 164 pounds John : 190 pounds

George : 180 pounds Young : 160 pounds

Now, let consider the linguistic proposition "fat male students". The students who belong to "fat male students" then constitute a fuzzy set, A. Is Hans ϵ A, John ϵ A, George ϵ A or Young ϵ A?

One plots the weights on a real line (see Figure 2.4) in order to present the relative differences. According to

common sense, a male student weighing more than 185 pounds at 5'9" height is absolutely considered a fat male student (actually The term "fat" depends on culture, race, etc. which are beyound our research). On the other hand, a male student weighing less then 160 pounds is not fat at all. Therefore, it is obvious that John is absolutely fat and Young is absolutely not fat. How about Hans and George? Actually Hans approaches Young with respect to weight and George is close to John. It is true that the heavier the male student is, the greater the degree to which he belongs to fuzzy set A. Thus, a degree-scaled line (see Figure 2.4) can be drawn corresponding to the previous weight-scaled line in order to represent the degree of membership indicating that a male student belongs to A. The scale on the degree-scaled line is linearly proportional to the weight-scaled line when the weight belongs to the interval [160, 185], As a result, the following grades of membership are available:

$$\text{degree}(\text{Hans} \in A) = \mu(x=H) = 0.2$$

$$\text{degree}(\text{John} \in A) = \mu(x=J) = 1$$

$$\text{degree}(\text{George} \in A) = \mu(x=G) = 0.8$$

$$\text{degree}(\text{Young} \in A) = \mu(x=Y) = 0,$$

where $\mu(.)$ (detailed definition is given in the next section) is the membership function of the fuzzy subset A of the set X.

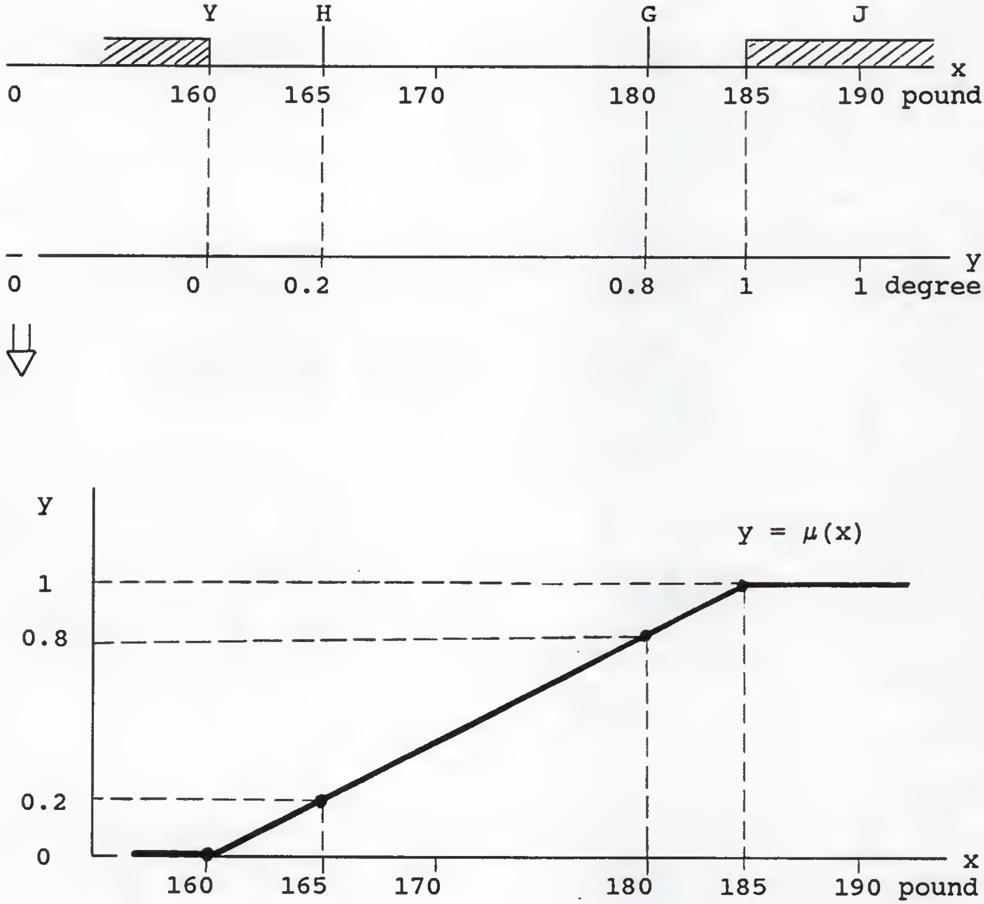


Figure 2.4. Derived degree of membership for the fuzzy set A

2.2.2 Fuzziness and Randomness

In order to be able to distinguish fuzziness from randomness, let us consider the following two examples:

Example [116]

Let us consider the statement "Hans eats x eggs for breakfast with $x \in A$, where $A = \{1, 2, ...\}$. One may get a possibility distribution (one of the most important characteristics of fuzzy sets theory) associated with A by interpreting $\mu_A(x)$ as the degree of ease with which Hans can eat x eggs. Also, it may have a probability distribution associated with A by interpreting $p_A(x)$ as the probability of Hans eating x eggs for breakfast. By some criterion, the values of $\mu_A(x)$ and $p_A(x)$ could be shown in Table 2.2. It shows that the possibility that Hans can eat 3 eggs is 1. However, the probability that Hans does so is rather small, e.g. 0.1.

Table 2.2. The possibility and probability of Hans eating x eggs[116]

x	1	2	3	4	5	6	7	8
$\mu_A(x)$	1	1	1	1	0.8	0.6	0.4	0.2
$p_A(x)$	0.1	0.8	0.1	0	0	0	0	0

The example clearly indicates that the fuzziness means the subjective criteria. This is how many eggs Hans can eat (ability). On the other hand, the stochastic measure considered a probability distribution which shows past results.

Example

The decision maker may feel that: "around $20,000 profit is acceptable"; "the budget should be around $9,000"; "dividends should be not higher than 6%"; "overtime should be less than 5% of the regular man hours" and so on. Obviously, these linquistic statements cannot be described by probability. The fuzzy sets theory, on the other hand, gives us a way to handle such linguistic situations.

2.2.3 The Representation of Fuzzy Sets

Consider a crisp set A = { x | x = 2y, y is natural number (N) }[13]. Obviously, A is a set of all even natural numbers. Thus, any natural number $y \in N$ is either belonging to A if it is even, or not belonging to A if it is odd. The crisp set A can then represented as:

$$A = \{ (1, 0), (2, 1), (3, 0), (4, 1), \ldots \}$$

where the second number of the ordered pairs, 0 or 1, is the measure of membership. 0 indicated that the first number of the ordered pairs is not even, and 1 for the even. Using this notation, one can apply it to the previous example about the proposition "fat student" and get the representation of the fuzzy set A as

$$A = \{ (\text{Hans}, 0.2), (\text{John}, 1), (\text{George}, 0.8), (\text{Young}, 0) \},$$

where the second number is no longer just 0 and 1, but in [0, 1]. This is the main characteristic differentiating the fuzzy sets from the classical (crisp) sets. Now let us consider the following definition.

Definition [24] Let X be a classical set of objects, called the universe, whose generic elements are denoted by x. The membership in a crisp subset of X is often viewed as characteristic function μ_A from X to { 0, 1 } such that

$$\begin{aligned} \mu_A(x) &= 1 \qquad \text{if and only if } x \in A \\ &= 0 \qquad \text{if and only if } x \notin A \end{aligned} \tag{2.2}$$

where { 0, 1 } is called a valuation set.

If the valuation set is allowed to be the real interval [0, 1], A is called a fuzzy set proposed by Zadeh [112]. $\mu_A(x)$ is the degree of membership of x in A. The closer the value of $\mu_A(x)$ is to 1, the more x belongs to A. Therefore, A is completely characterized by the set of ordered pairs

$$A = \{ (x, \mu_A(x)) \mid x \in X \}. \tag{2.3}$$

It is worth noting that the characteristic function is also called a possibility distribution or a membership function. In this study, membership function is preferred to others.

A more convenient notation was proposed by Zadeh [109].

1. When X is a finite set { x_1, x_2, ..., x_n }, a fuzzy set A is then expressed as

$$\begin{aligned} A &= \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n \\ &= \Sigma_i \ \mu_A(x_i)/x_i. \end{aligned} \tag{2.4}$$

For the previous example, A can be expressed as

$$\begin{aligned} A &= \mu_A(\text{Hans})/\text{Hans} + \mu_A(\text{John})/\text{John} + \mu_A(\text{George})/\text{George} \\ &\quad + \mu_A(\text{Young})/\text{Young} \\ &= 0.2/\text{Hans} + 1/\text{John} + 0.8/\text{George} + 0/\text{Young}. \end{aligned}$$

2. When X is not a finite set, A then can be written as

$$A = \int_X \mu_A(x)/x \tag{2.5}$$

In fuzzy sets theory, the following three concepts are important:

Support[1]. The support of a fuzzy set A is the crisp subset of X and is presented as:

$$\text{supp } A \; \{ \; x \mid \mu_A(x) > 0 \text{ and } x \in X \; \}$$

For the previous example, supp A = { Hans, John, George }.

α-level set (α cut)[1]. The α-level set (α cut) of a fuzzy set A is a crisp subset of X and is denoted by

$$A_\alpha = \{ \; x \mid \mu_A(x) \geq \alpha \text{ and } x \in X \; \}.$$

For the previous example, $A_{0.2}$ = { Hans, John, George } and $A_{0.7}$ = { John, George }.

Convexity. A fuzzy set A in X is (strong) fuzzy-convex if and only if for every pair of the points x^1 and x^2 in X, the membership function of A satisfies the following inequality:

$$\mu_A(\delta x^1 + (1 - \delta)x^2) \geq \min\,(\mu_A(x^1),\, \mu_A(x^2)),$$

$$(\; \mu_A(\delta x^1 + (1 - \delta)x^2) > \min\,(\mu_A(x^1),\, \mu_A(x^2)),\;)$$

where $\delta \in [0, 1]$.

Since we are emphasizing fuzzy linear programming problems, it is necessary to consider the definition of fuzzy decision proposed by Bellman and Zadeh [1]. They assumed that objective(s) and constraints in an imprecise situation can be

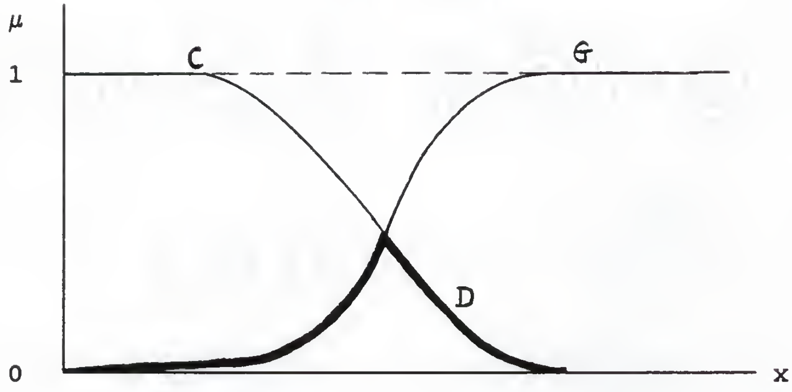


Figure 2.5. The relation of the sets of G, C and D

represented by fuzzy sets. A decision, then, may be stated as the confluence of objective(s) and constraints, and may be defined as follows.

Definition[1]. Assume that we are given a fuzzy objective set G and a fuzzy constraint set C in a space of alternatives X. Then, G and C combine to form a decision D which is a fuzzy set resulting from intersection of G and C, and correspondingly $\mu_D = \mu_G \wedge \mu_C$ (A: conjunction). The relation between G, C and D is depicted in Figure 2.5.

A maximizing decision then can be defined as follows:

$$\mu_D(x^M) = \max \mu_D(x), \qquad x \in X$$
$$= 0 \qquad \text{elsewhere.} \qquad (2.6)$$

If $\mu_D(x)$ has a unique maximum at x^M, then the maximizing decision is a uniquely defined crisp decision which can be

interpreted as the action which belongs to all fuzzy sets representing either constraints or objective(s) with the highest possible degree of membership.

2.3 Fuzziness in Linear Programming

In order to understand as clearly as possible how and where the fuzziness is applied in linear programming problems, a simplified linear programming case is demonstrated.

Example

The Hardee toy company makes two kinds of toy dolls. Doll A is a high quality toy with $0.40 unit profit and doll B is of lower quality with $0.30 unit profit. Each doll A requires twice as many labor hours as each doll B, and the total available labor hours are 500 hours per day. The supply of material is sufficient for only 400 dolls per day (both A and B combined). It is assumed that all the dolls the factory has made can be sold. The manager, then, wishes to maximize the total profit in scheduling the production.

Since the manager (who got a MS degree in Industrial Engineering) does very well in linear programming, he formulates the production scheduling problem as follows:

$$\begin{array}{lll} \text{maximize} & Z = 0.4x_1 + 0.3x_2 & \text{(profit)} \\ \text{subject to} & g_1(x) = x_1 + x_2 \le 400 & \text{(materials)} \\ & g_2(x) = 2x_1 + x_2 \le 500 & \text{(labor hours)} \\ & x_1,\ x_2 \ge 0 & \end{array}$$

where x_1 is the amount of doll A to be produced and x_2 is the amount of doll B to be produced. After solving the above LP problem by using the graphic method (see Fig.2.6), the manager gets the optimal solution: he produces 100 units of doll A and

300 units of doll B, and earns \$130 profit. Subsequently, the manager feels that the available material may change in some range because he can get additional materials from suppliers, and that available time also may change because he can ask the workers to work overtime. Therefore, he begins studying fuzzy sets theory and introduces it into his production scheduling problem. Now the manager decides that the profit should be substantially larger than \$130 because of the increased resources.

The possibility distributions (membership functions) of the material, time and profit shown in Table 2.3 and Figure 2.7 are provided by the manager. He explains that the

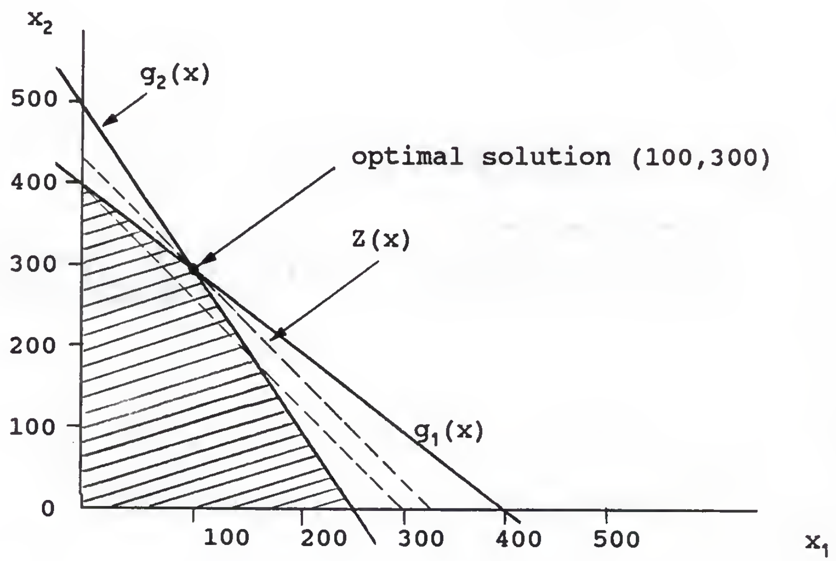


Figure 2.6. The graphic approach for a linear programming problem

values of the possibilities mean the degree of his satisfaction on the constraints g_1 (material) and g_2 (labor hours), and the objective Z. For example, when the total amount of used material is less than or equal to 400, the constraint " $g_1(x) \leq 400$ " is absolutely satisfied. When the total amount of used material is greater than or equal to 500, the constraint " $g_1(x) \leq 400$ " is absolutely violated, so the degree of satisfaction is equal to 0. Between 400 and 500, the degree of membership function is monotonic linear decreasing.

Table 2.3. The possibility distributions of the material, labor hours, and profit

material	g_1	150	200	250	300	350	400	450	500	500
available	μ_{g1}	1	1	1	1	1	1	0.5	0	0
time	g_2	200	250	300	350	400	450	500	550	600
available	μ_{g2}	1	1	1	1	1	1	1	0.5	0
	Z	80	90	100	110	120	130	140	150	160
profit	μ_z	0	0	0	0	0	0	0.33	0.67	1

The other two possibilities can be explained in the same way. Thus, the manager considers that the only feasible solutions should be those satisfying the possibility distributions of the material, labor hours, and profit at the same time. The feasible alternative with maximum degree of satisfaction is then chosen as the optimal solution of

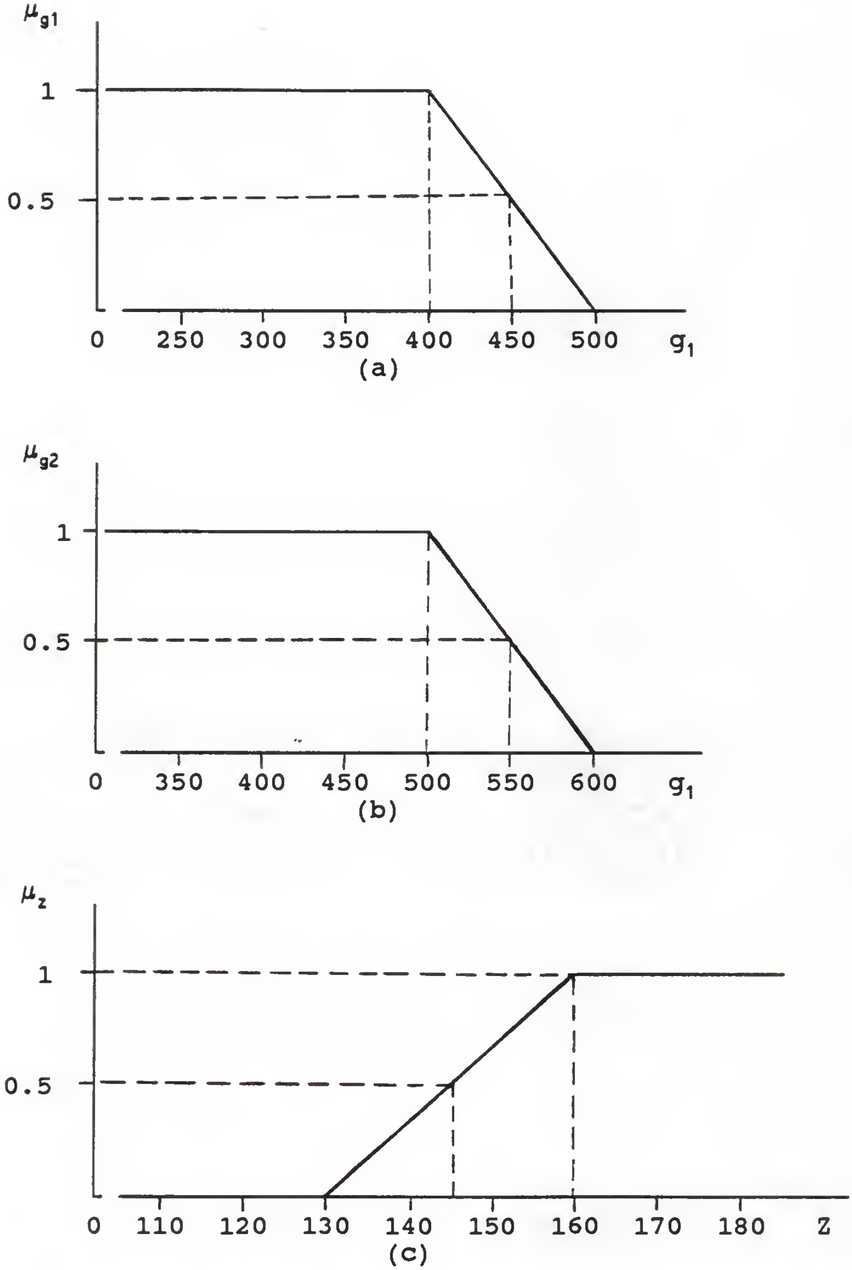


Figure 2.7. The possibility distributions (membership functions) of the material, labour hours, and profit

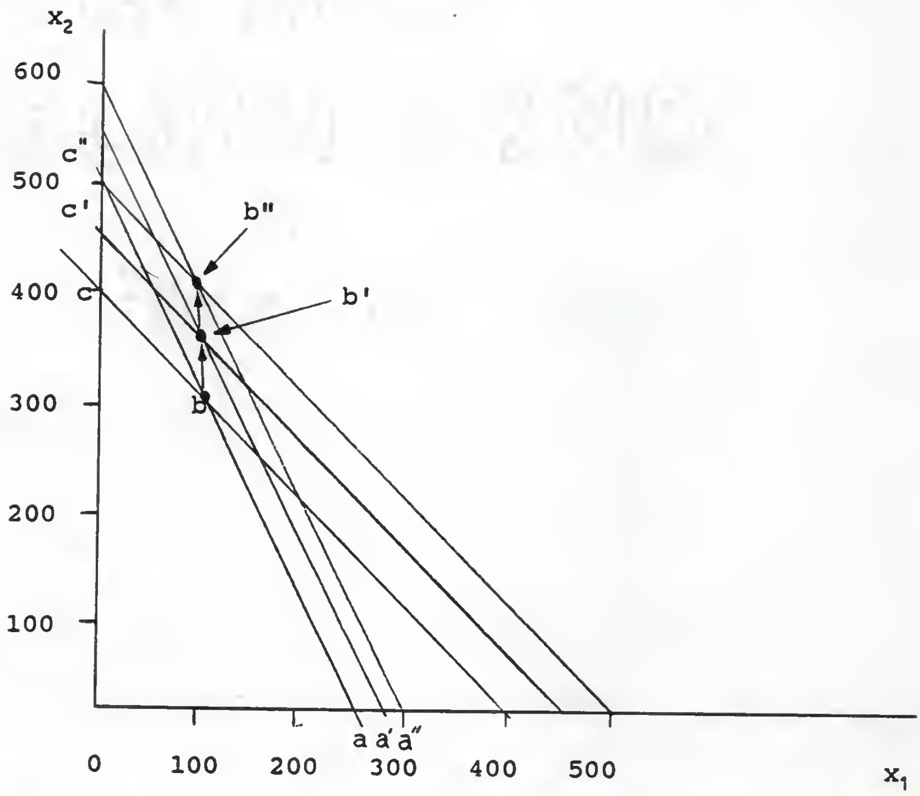


Figure 2.8. The graphic solution

the production scheduling problem. Finally, from Figure 2.8, when the available material and labor hours are 450 and 550, respectively, the optimal solution point b' is Z' = 145, $x_1 = 100$ and $x_2 = 350$, and the overall degree of satisfaction is 0.5. At point b, $g_1(x) = 400$, $g_2(x) = 500$, Z = 130, $x_1 = 100$, $x_2 = 300$, the total degree of satisfaction is one. At point b", $g_1(x) = 500$, $g_2(x) = 600$, Z" = 160, $x_1 = 100$ and $x_2 = 400$. The total degree of satisfaction is equal to zero.

From the above case, it is obvious that in fuzzy linear

programming, the fuzziness of the constraints and objective function is characterized by the possibility distributions over the tolerance range (ambiguous range). Therefore, fuzzy linear programming is used to study an optimization system which is characterized by the fuzzy objective and/or fuzzy constraints over given tolerance ranges characterized by their corresponding membership functions. The optimal solution is the one which maximizes the membership grades of the fuzzy objective and fuzzy constraints at the same time.

Since Bellman and Zadeh [1] proposed min-operator for solving fuzzy mathematical programming in 1970, there have appeared a large variety of methodologies in the literature. Because of the fact that no approach can satisfy both possible fuzzy environments and requirements of decision makers, a clear classification is needed in order to be able to use an appropriate approach to fit any possible situation (membership function). Figure 2.9 displays a taxonomy of fuzzy linear programming. According to Zimmermann's [130] classification, there are two major classes of fuzzy linear programming: symmetric and nonsymmetric models. A symmetric model is designed to solve the problems with a fuzzy objective and fuzzy constraints. A nonsymmetric model is used to solve the problems with a fuzzy objective and crisp constraints, or a crisp objective and fuzzy constraints. For each model (for example, a symmetric model), the final optimal solutions can be crisp solutions or fuzzy solutions.

Now, let us go into the detailed review of these approaches.

FUZZY LINEAR PROGRAMMING

SYMMETRIC MODELS

Crisp Solution

- Method of Zimmermann
- Parametric Approaches
  - nontradeoff models
  - tradeoff models
  - triangular fuzzy number

Fuzzy Solution:

- A Parametric Approach

NONSYMMETRIC MODELS

Crisp Solution:

- α-level set approach
- an equivalent symmetric model
- L-R fuzzy number

Fuzzy Solution:

- Parametric Approaches
  - fuzzy constraints
  - fuzzy objective

Figure 2.9. A taxonomy of fuzzy linear programming

2.4 The Literature Survey of Fuzzy Linear Programming

In this section, we will discuss the current approaches to fuzzy linear programming in the problem-oriented manner. These approaches were all based on the min-operator proposed by Bellman and Zadeh in 1965 and 1970. Even though the argument of the appropriateness of the min-operator in presenting the real-world problems is high, it is still considered as the most important operator in fuzzy mathematical programming and fuzzy decision making problems. Bellman and Giertz [2] argued from a logic point of view, interpreting the intersection as "logical and", the union as "logical or" and the fuzzy set A as the statement. For an axiomatic justification, Bellman and Giertz considered: two statements, S and T, for which the truth values are $\mu(S)$ and $\mu(T)$, respectively. $\mu(S)$ and $\mu(T) \in [0, 1]$. The truth values of the "and" and "or" combination of these two statements are $\mu(S \text{ and } T)$ and $\mu(S \text{ or } T)$, which belong to [0, 1] and are interpreted as the values of the membership functions of the intersection and the union, respectively, of S and T statements.

Bellman and Giertz assumed that whenever truth values have been assigned to (arbitrary) statements S and T, then two real-valued function f and g provide us with truth values for "S and T" and "S or T" such that

$$\mu(S \text{ and } T) = f[\mu(S), \mu(T)],$$

$$\mu(S \text{ or } T) = g[\mu(S), \mu(T)]. \tag{2.7}$$

They feel that the following restrictions are reasonably imposed on f and g. The original statements are shown as follows:

(i) f and g are nondecreasing and continous in both variables. Our willingness to accept "S and T" or to accept "S or T", should not decrease if our willingness to accept S(or T) increases. With suitably small changes in the level of acceptance of S or T we should be able to change the truth value of "S and T", or of "S or T". by an arbitrarily small amount.

(ii) f and g are symmetric, i.e. $f(x, y) = f(y, x)$ and $g(x, y) = g(y, x)$. There is no reason to assign different truth values to "S and T" and "T and S".

(iii) $f(x, x)$ and $g(x, x)$ are strictly increasing in x. If $\mu(S_1) = \mu(S_2) > \mu(S_3) = \mu(S_4)$ we should be more willing to accept "S_1 and S_2" than to accept "S_3 and S_4".

(iv) $f(x, y) \le \min\{x, y\}$ and $g(x, y) \ge \max\{x, y\}$. Accepting "S and T" requires more, and accepting "S or T" less, than accepting S or T alone. Thus $\mu(S \text{ and } T) \le \mu(S)$ and so on.

(v) $f(1, 1) = 1$ and $g(0, 0) = 0$. If both S and T are completely accepted, we must accept "S and T" completely as well, and if S and T are both completely rejected, then we must also reject "S or T".

(vi) Logically equivalent statements have equal truth values. To illustrate with an example, the statement

S_1 and (S_2 or S_3)

is logically equivalent to

(S_1 and S_2) of (S_1 and S_3)

we have no reason to be more willing to accept one of these statements over the other. Condition (ii) is another example of this principle.

Bellman and Giertz then proved mathematically that

$$\mu(S \text{ and } T) = \min\{\mu(S), \mu(T)\},$$

$$\mu(S \text{ or } T) = \max\{\mu(S), \mu(T)\}. \tag{2.8}$$

Let us divide the variety of fuzzy linear programming problems into six main problems as follows:

Problem 1.

$$
\begin{array}{lll}
\widetilde{\text{maximize}} & Z = c^T x & \\
\text{subject to} & (Ax)_i \underset{\sim}{\leq} b_i, & i = 1, 2, \ldots, m, \\
 & x \geq 0 & \qquad (2.9)
\end{array}
$$

For problem 1, there are two approaches proposed by Zimmermann in 1976 and Chanas in 1983.

(i) Zimmermann's Approach[123].

In this approach, the goal b_0 and its corresponding tolerance p_0 of the fuzzy objective are given initially, and so are the fuzzy resources: b_i and its corresponding tolerances p_i, $\forall$ i. The fuzzy objective and the fuzzy constraints are then considered without difference, and their corresponding regions can be described in the intervals $[b_i, b_i + p_i]$, $\forall$ i. Thus, Problem 1 can be considered as

$$
\begin{array}{ll}
\text{find} & x \\
\text{such that} & c^T x \underset{\sim}{\geq} b_0 \\
 & (Ax)_i \underset{\sim}{\leq} b_i, \quad \forall\, i, \\
 & x \geq 0 \qquad (2.10)
\end{array}
$$

In fuzzy sets theory, the fuzzy objective function and the fuzzy constraints are defined by their corresponding membership functions. For simplicity, let us assume that the membership function μ_0 of the fuzzy objective is non-

decreasing continuous linear function, and the membership functions μ_i, $\forall$ i, of the fuzzy constraints are non-increasing continuous linear membership functions as follows:

$$\mu_0 = \begin{cases} 1 & \text{if } c^T x > b_0 \\ 1 - \dfrac{b_0 - c^T x}{p_0} & \text{if } b_0 - p_0 \le c^T x \le b_0 \\ 0 & \text{if } c^T x < b_0 - p_0 \end{cases} \qquad (2.11)$$

(see Figure 2.10).

$$\mu_i = \begin{cases} 1 & \text{if } (Ax)_i < b_i \\ 1 - \dfrac{(Ax)_i - b_i}{p_0} & \text{if } b_i \le (Ax)_i \le b_i + p_i \\ 0 & \text{if } (Ax)_i > b_i + p_i,\ \forall\ i \end{cases} \qquad (2.12)$$

(see Figure 2.11)

Zimmermann, then, applied the min-operator concept to solve the equation (2.10). Thus, the optimal solution will exist at

$$\text{maximize } \mu_D = \max\ [\ \min\ (\mu_0, \mu_1, \ldots, \mu_m)\ ] \qquad (2.13)$$

where μ_D is the membership function of the decision space D, and $\mu_D = \min\ (\mu_0, \mu_1, \ldots, \mu_m)$.

Now, let us assume $\lambda = \mu_D = \min\ (\mu_0, \mu_1, \ldots, \mu_m)$, the

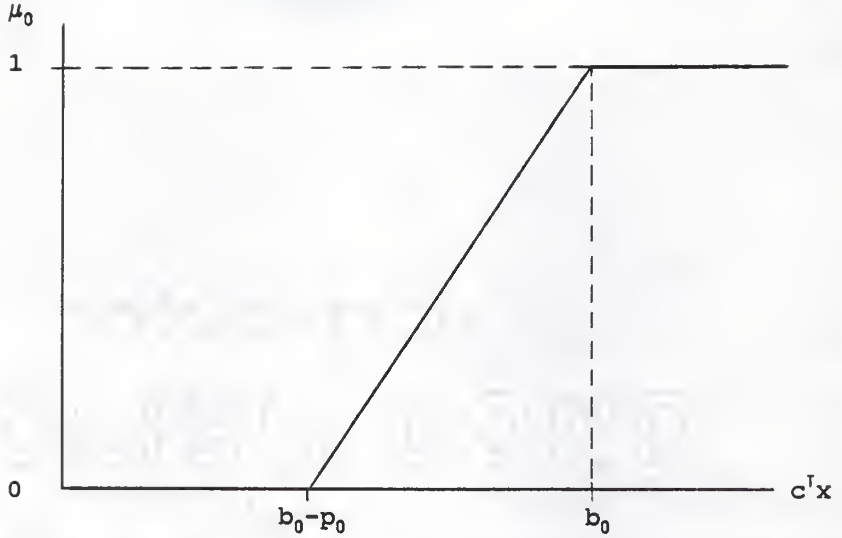


Figure 2.10. The membership function of the fuzzy objective constraint $c^Tx \gtrsim b_0$

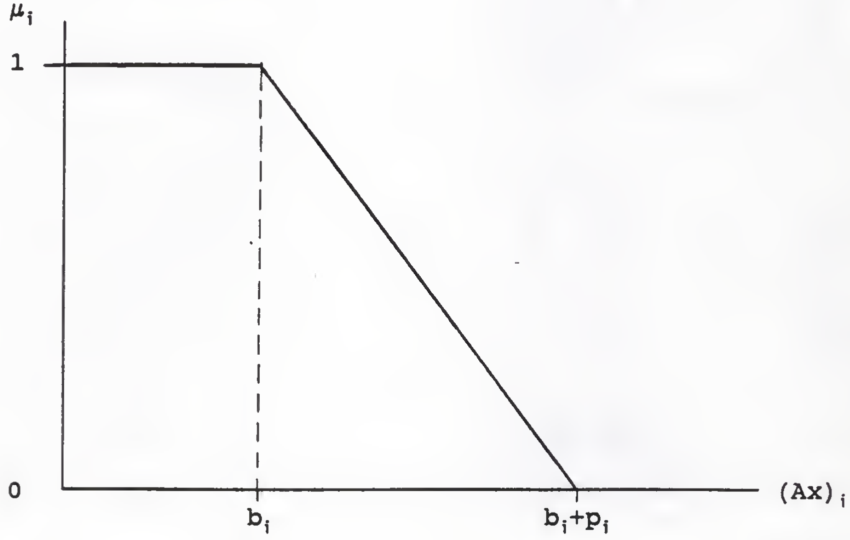


Figure 2.11. The membership function for the ith fuzzy constraint $(Ax)_i \lesssim b_i$

problem given by (2.10), via the equation (2.13), is equivalent to

$$\begin{aligned} &\text{maximize} && \lambda \\ &\text{subject to} && \mu_0 = 1 - (b_0 - c^T x)/p_0 \geq \lambda \\ &&& \mu_1 = 1 - [(Ax)_1 - b_1]/p_1 \geq \lambda \\ &&& \vdots \\ &&& \mu_m = 1 - [(Ax)_m - b_m]/p_m \geq \lambda \\ &&& \lambda, \mu_i, \forall\ i, \in [0, 1], \end{aligned} \tag{2.14}$$

or

$$\begin{aligned} &\text{maximize} && \lambda \\ &\text{subject to} && c^T x \geq b_0 - (1 - \lambda) p_0 \\ &&& (Ax)_i \leq b_i + (1 + \lambda) p_i, \quad \forall\ i, \\ &&& x \geq 0 \text{ and } \lambda \in [0, 1], \end{aligned} \tag{2.15}$$

where c, A, b_0, p_0, b_i and p_i, $\forall$ i, are given initially. (2.15) then can be solved by a traditional programming technique. A unique optimal solution is obtained.

(ii) A Parametric Approach Using A Nontradeoff model -- Chanas Approach [10]

For Problem 1, Chanas considered that the goal b_0 and its tolerance p_0 cannot be specified initially because of lack of knowledge about the fuzzy feasible region. Therefore, in order to help the decision maker to determine b_0 and p_0, Chanas first solves the following problem, and then present the

results to the decision maker to determine b_0 and p_0:

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & (Ax)_i \lesssim b_i, \quad \forall\ i, \\ & x \geq 0. \end{array} \tag{2.16}$$

When c, A, b_i and p_i, $\forall$ i, are given, (2.16) will be equivalent to the following parametric linear programming:

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & (Ax)_i \leq b_i + \theta p_i, \quad \forall\ i, \\ & x \geq 0 \text{ and } \theta \in [0, 1], \end{array} \tag{2.17}$$

where c, A, b_i and p_i, $\forall$ i, are given and θ is a parameter. The optimal solutions $Z^*(\theta)$ and $x^*(\theta)$ are then presented to the decision maker. The decision maker now can choose b_0 and its corresponding p_0. According to these values, we can then construct the membership function μ_0 of the objective function as (2.11). Since the final optimal solution will exist at $x^*(\theta)$, μ_0 becomes

$$\mu_0(x^*(\theta)) = \begin{cases} 1 & \text{if } c^T x^* > b_0 \\ 1 - \dfrac{b_0 - c^T x^*}{p_0} & \text{if } b_0 - p_0 \leq c^T x^* \leq b_0 \\ 0 & \text{if } c^T x^* < b_0 - p_0 \end{cases} \tag{2.18}$$

(see Figure 2.12)

which is always piecewise linear function with respect to θ.

In (2.17), the overall membership function μ_c of the fuzzy constraint set is $1 - \theta$, by use of min-operator. Therefore, the final optimal solution $x^*(\theta^*)$ and $Z^*(\theta^*)$ will exist at

$$\max \mu_D = \max [\min (\mu_0, \mu_c)] = \max (\mu_0 \wedge \mu_c) \quad (2.19)$$

as depicted in Figure 2.12.

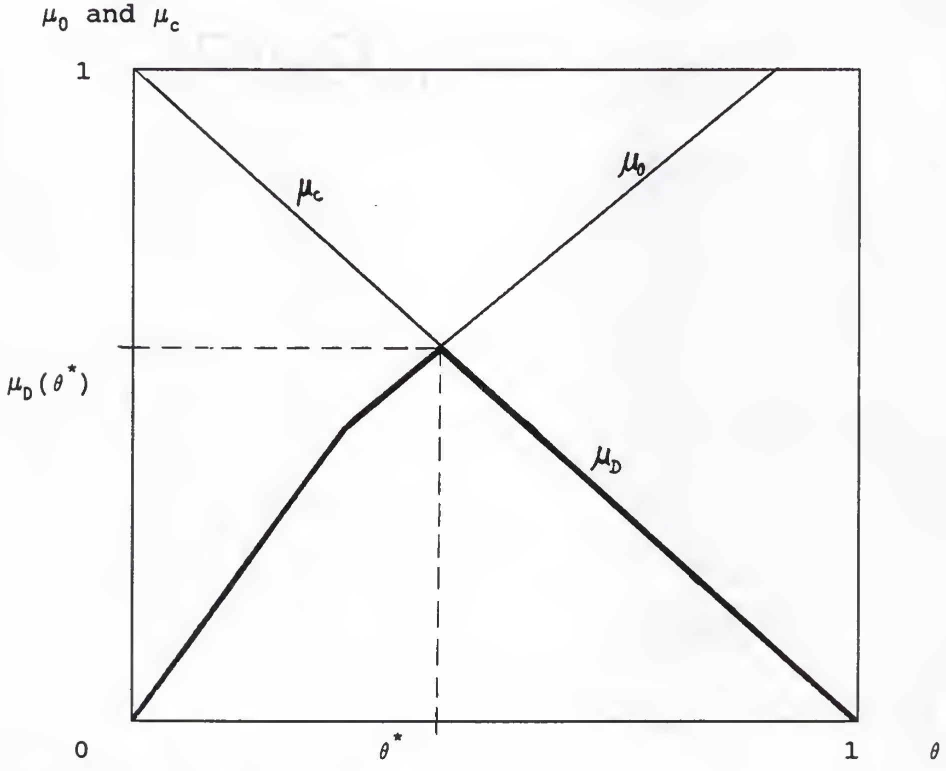


Figure 2.12. The intersection of μ_0 and μ_c.

Problem 2.

$$\widetilde{\text{maximize}} \quad \tilde{c}^T x$$

$$\text{subject to} \quad (\tilde{A}x)_i \lesssim \tilde{b}_i, \qquad \forall\ i,$$

$$x \geq 0, \qquad (2.20)$$

For Problem 2, there are two approaches proposed by Carlsson and Korhonen (1986), and Tanaka and Asai (1984).

(i) A Parametric Approach Using A Tradeoff Model -- Carlsson et al. [9]

For a traditional linear programming model (2.1), Carlsson et al. assume that the decision maker can specify the intervals $[c^0, c^1)$, $[A^0, A^1)$, and $[b^0, b^1)$ for the possible values of the parameters c, A and b. At the same time, they established a relationship between a solution in the model (2.1) and its parameters: the solution $Z^* = Z^*(c, -A, b)$ of the model (2.1) is an increasing function of the parameter c, -A and b. Thus, we can reasonably assume that membership functions are monotonically decreasing functions of the parameters, c, -A, b as shown in Figure 2.13 where exponential forms are assumed. That is:

$$\mu_c = a_c[1 - \exp\{-b_c(c - c^1)/(c^0 - c^1)\} \qquad (2.21)$$

where $b_c > 0$ or $b_c < 0$, and $a_c = 1/(1 - \exp\{-b_c\})$ and $c \in [c^0, c^1]$. $\mu_c = 1$ when $c \leq c^0$, $\mu_c = 0$ when $c > c^1$. b_c is specified by the decision maker. In the same manner, the membership function of A and b can be obtained.

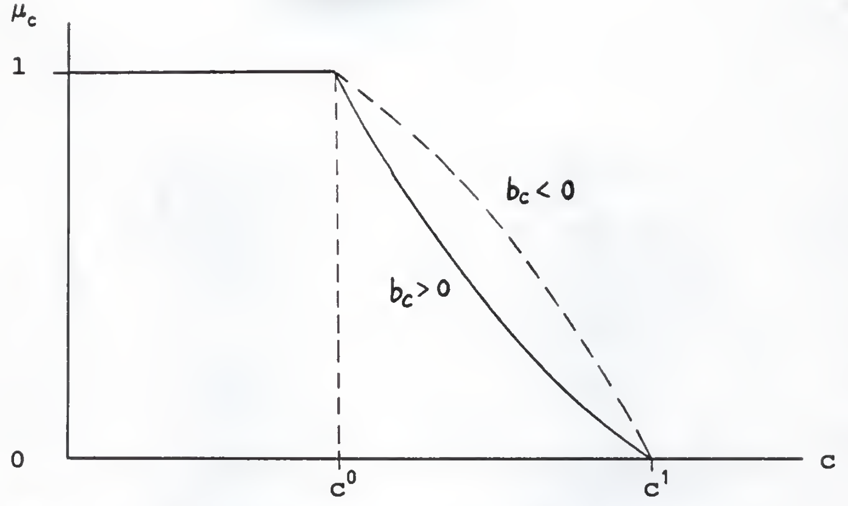


Figure 2.13. The membership function of c

After full tradeoff between c, -A and b, the solution will always exist at

$$\mu = \mu_c = \mu_A = \mu_b. \quad (2.22)$$

Therefore, we can obtain the following equation:

$$c = g_c(\mu)$$

$$A = G_A(\mu)$$

$$b = g_b(\mu), \quad (2.23)$$

where μ = [0, 1] and g_c, G_A and g_b are inverse functions of μ_c, μ_A and μ_b. Thus, the fuzzy linear programming problem (2.20) becomes:

$$\begin{aligned} &\text{maximize} && [g_c(\mu)]^T x \\ &\text{subject to} && [G_A(\mu)]x \le g_b(\mu) \\ & && x \ge 0. \end{aligned} \tag{2.24}$$

Obviously, (2.24) is a nonlinear programming problem. However, it can be solved by any linear programming technique if μ is given. Thus, we can obtain a set of solutions corresponding to a set of μs and then plot these solution pairs (Z^*, μ). After referring to this relationship, the decision maker can choose his solution for implementation.

(ii) A Fuzzy Function System With Triangular Fuzzy Numbers -- Tanaka et al. [88]

Tanaka et al. considered the problem given by (2.20) as:

$$\begin{aligned} &\text{find} && x \\ &\text{such that} && \tilde{Y}_0 = -\tilde{B}_0 + \tilde{A}_{01}x_1 + \ldots + \tilde{A}_{0n}x_n \gtrsim 0 \\ & && \tilde{Y}_1 = \tilde{B}_1 - (\tilde{A}_{11}x_1 + \ldots + \tilde{A}_{1n}x_n) \gtrsim 0 \\ & && \vdots \\ & && \tilde{Y}_m = \tilde{B}_m - (\tilde{A}_{m1}x_1 + \ldots + \tilde{A}_{mn}x_n) \gtrsim 0 \end{aligned} \tag{2.25}$$

Where $\tilde{B}_0 = (b_0, p_0)$, $\tilde{B}_i = (b_i, p_i)$, $\tilde{A}_{0j} = \tilde{c}_j = (a_{0j}, d_{0j})$ and $\tilde{A}_{ij} = (a_{ij}, d_{ij})$, $\forall$ i and j, are given. The first numbers of the ordered pairs are the centers of the triangular membership functions and the second numbers expressing the fuzziness are the deviations from the corresponding centers. These

triangular membership functions $\mu_{\tilde{B}i}$ and $\mu_{\tilde{A}ij}$, $\forall$ i and j, of the fuzzy parameters are described as follows (see Figure 2.14 and Figure 2.15):

$$\mu_{\tilde{B}i}(B_i) = \begin{cases} 1 - \dfrac{|B_i - b_i|}{p_i} & \text{if } b_i - p_i \leq B_i \leq b_i + p_i \\ 0 & \text{otherwise,} \end{cases} \quad (2.26)$$

$$\mu_{\tilde{A}ij}(Ai_j) = \begin{cases} 1 - \dfrac{|A_{ij} - a_{ij}|}{d_{ij}} & \text{if } a_{ij} - d_{ij} \leq A_{ij} \leq a_{ij} + d_{ij} \\ 0 & \text{otherwise} \end{cases} \quad (2.27)$$

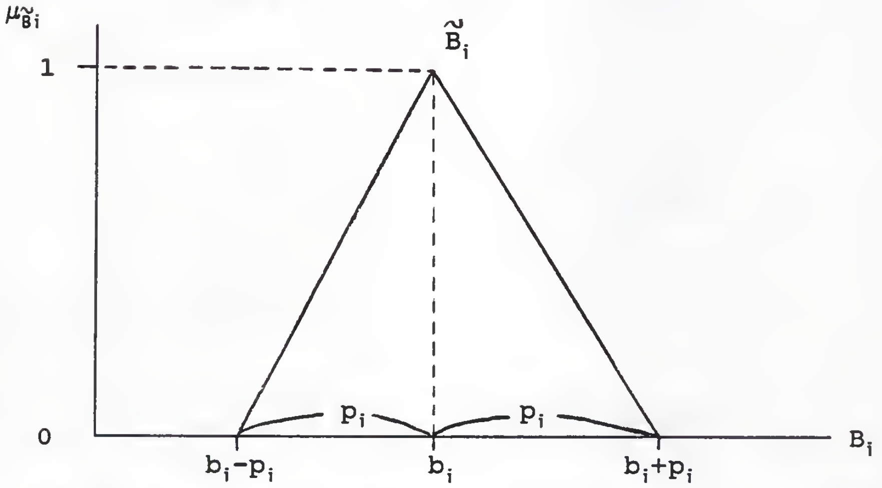


Figure 2.14. The membership functions of $\tilde{B}_i$, for i = 0, 1, 2, ..., m

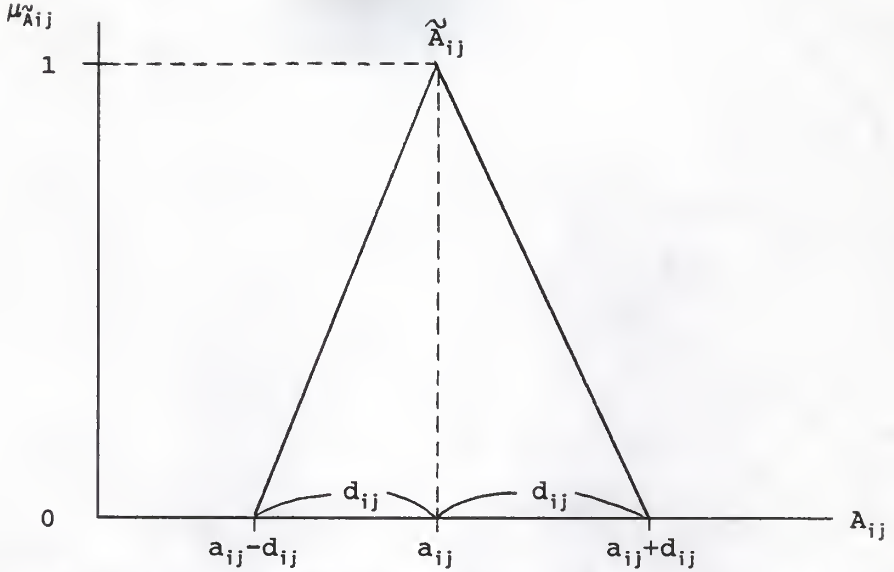


Figure 2.15. The membership functions of $\tilde{A}_{ij}$, $i = 0, 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$

for $i = 0, 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Tanaka et al. presented the following proposition and definition:

Proposition. $\tilde{Y}_i$, $\forall$ i, are defined by their membership functions such as:

$$\mu_{Y0}(y_0) = \begin{cases} 1 - \dfrac{|y_0 - (\Sigma_j a_{0j}x_j - b_0)|}{\Sigma_j d_{0j}x_j + p_0} & \text{if } x \neq 0 \\ 1 & \text{if } x = y = 0 \\ 0 & \text{if } x = 0,\ y \neq 0 \end{cases} \tag{2.28}$$

and

$$\mu_{Yi}(y_i) = \begin{cases} 1 - \dfrac{|y_i - (b_i - \Sigma_j a_{ij}x_j)|}{\Sigma_j d_{ij}x_j + p_i} & \text{if } x \neq 0 \\ 1 & \text{if } x = y = 0 \\ 0 & \text{if } x = 0, \; y \neq 0 \end{cases} \tag{2.29}$$

for i = 1, 2, ..., m. (see Figure 2.16)

Definition. "$\tilde{Y}_i$ is almost positive" denoted by $\tilde{Y}_i \gtrsim 0$ is defined by

$$\tilde{Y}_i \gtrsim 0 \iff \mu_{\tilde{Y}i}(0) \leq 1 - h_i, \; i = 0, 1, 2, \ldots, m$$

and

$$-b_0 + a_{01}x_1 + a_{02}x_2 + \ldots + a_{0n} \geq 0$$

$$b_i - a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in} \geq 0, \; i = 1, 2, \ldots, m \tag{2.30}$$

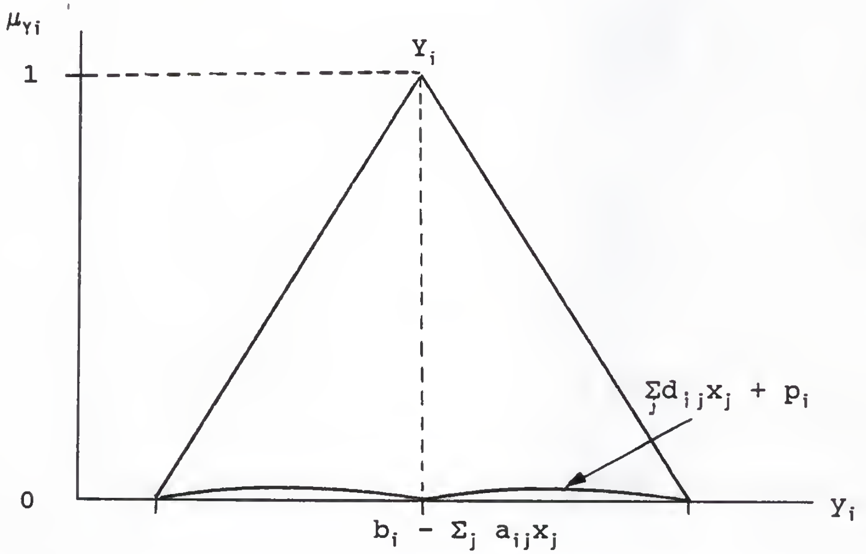


Figure 2.16. The membership functions of $\tilde{Y}_i$, i = 1, 2, ..., m

where $h_i \in [0, 1]$, $i = 0, 1, 2, \ldots, m$, stand for a degree of "$\tilde{Y}_i \gtrsim 0$" and the larger h_i is, the stronger the meaning of "almost positive" is. Figure 2.17 shows the relationship and the meaning. Let us consider:

$$S = \frac{(\text{positive area of } \tilde{Y}_i)}{(\text{total area of } \tilde{Y}_i)}$$

According to the similar triangular principle, we find $S = (1 - h)^2/2 = 0.5 + h - 0.5h^2$. If $h = 0.5$, we have $S = 0.875$. It means that Y can be said to be "almost positive with the grade 0.875 in a sense of the area".

From the results of the above definition and proposition, the fuzzy linear programming problem (2.25) will be equivalent to

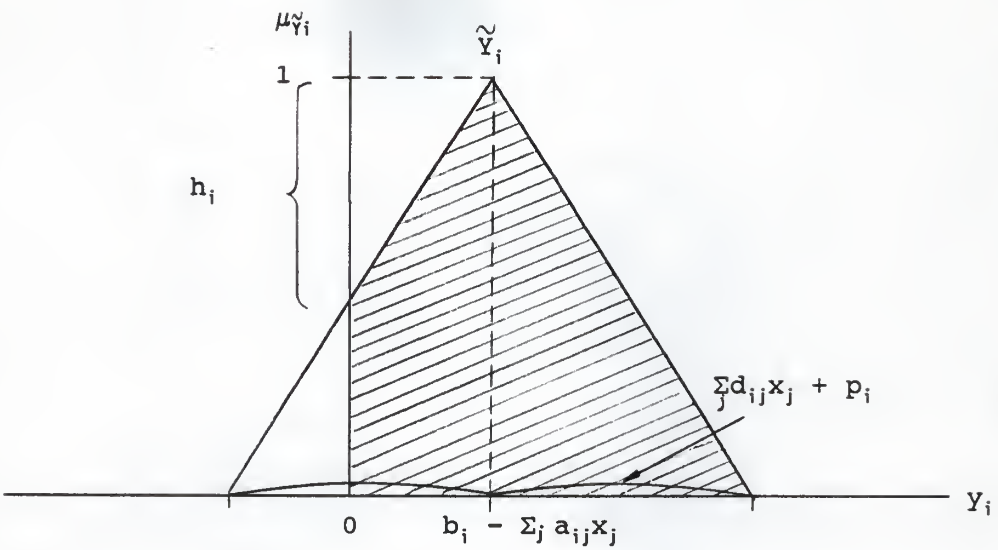
$$\begin{array}{ll} \text{find} & x \\ \text{such that} & \mu_{\tilde{Y}_i}(0) \leq 1 - h_i, \quad i = 0, 1, 2, \ldots, m \end{array} \tag{2.31}$$

where $\mu_{\tilde{Y}_i}(0)$ as defined in (2,28) and (2.29) when $y_i = 0$, $i = 0, 1, 2, \ldots, m$.

By use of min-operator concept, the optimal solution will exist at

$$\max\ [\ \min\ (h_0, h_1, \ldots, h_m)\ ] \tag{2.32}$$

Now, let us assume $h = \min\ (h, h_1, \ldots, h_m)$. Then, the equation (2.32) becomes:



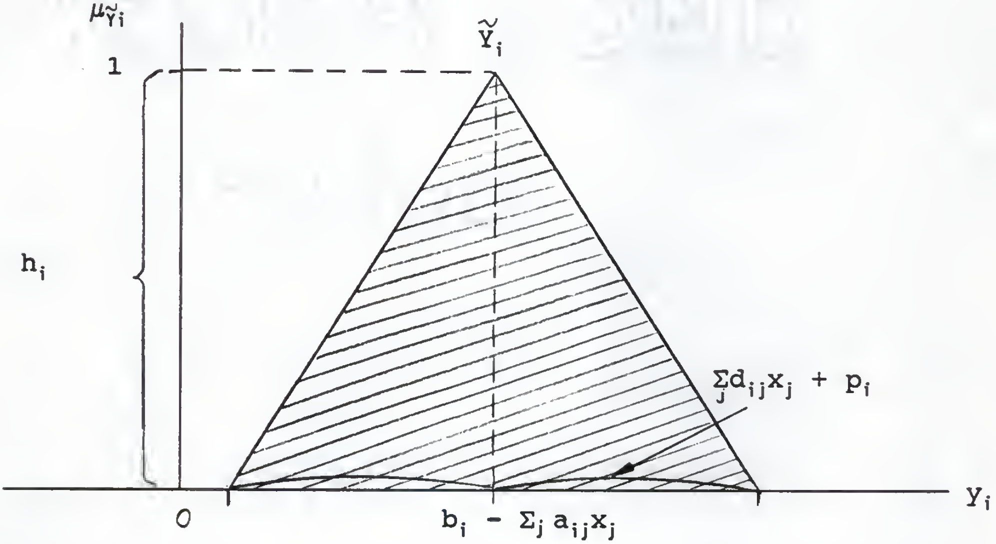


Figure 2.17. The relationship of h_i and "$\tilde{Y}_i \underset{\sim}{\geq} 0$", for i = 1, 2, ..., m

$$\begin{aligned}
\text{maximize} \quad & h \\
\text{subject to} \quad & \Sigma_j \, (a_{0j} - hd_{0j})x_j \geq (b_0 + hp_0) \\
& \Sigma_j \, (a_{ij} + hd_{ij})x_j \geq (b_i - hp_i), \\
& \qquad\qquad i = 1, 2, \ldots, m \\
& x \geq 0 \text{ and } h \in [0, 1]. \qquad (2.33)
\end{aligned}$$

The solution is then the solution of equation (2.25). Obviously, the problem given by (2.33) is a nonlinear programming problem. However, we can solve this problem by the following algorithm:

Algorithm.

Step 1. Determine a small initial value of h, so that there exists an admissible set satisfying (2.33).

Step 2. Let $\delta > 0$ be a small increment. Then, find $h + (k + 1)\delta$ where there is no admissable set satisfying (2.33) in the process of increasing h by δ, 2δ, ...

Step 3. Since there exists an admissable set in the level of $h + k\delta$, regard this set as the constraint set and choose the most interesting inequality from equation (2.33) as an objective function. Suppose that it is the sth inequality. The problem becomes to maximize the objective function $= a_{s1}x_1 + a_{s2}x_2 + \ldots + a_{sn}x_n$, subject to the same constraint set as (2.33), but $i \neq s$. Then, the optimal solution will be a good approximate solution of the problem of (2.25).

Problem 3.

$$\widetilde{\text{maximize}} \qquad Z = c^T\tilde{x}$$

$$\text{subject to} \qquad A\tilde{x} \mathrel{\tilde{\leq}} \tilde{b} \qquad (2.34)$$

where $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ and $\tilde{x}_j$, $\forall$ j, are assumed as triangular fuzzy numbers with the triangular membership function as shown in Figure 2.18, and the fuzzy numbers $\tilde{b} = (\tilde{b}_1, \ldots, \tilde{b}_m)$, and $\tilde{b}_i$, $\forall$ i, are also assumed as fuzzy triangular numbers described in the previous model. There is an approach proposed by Tanaka and Asai (1984).

A Parametric Approach -- Tanaka et al. [87]

Tanaka et al. considered the problem represented by (2.34) as

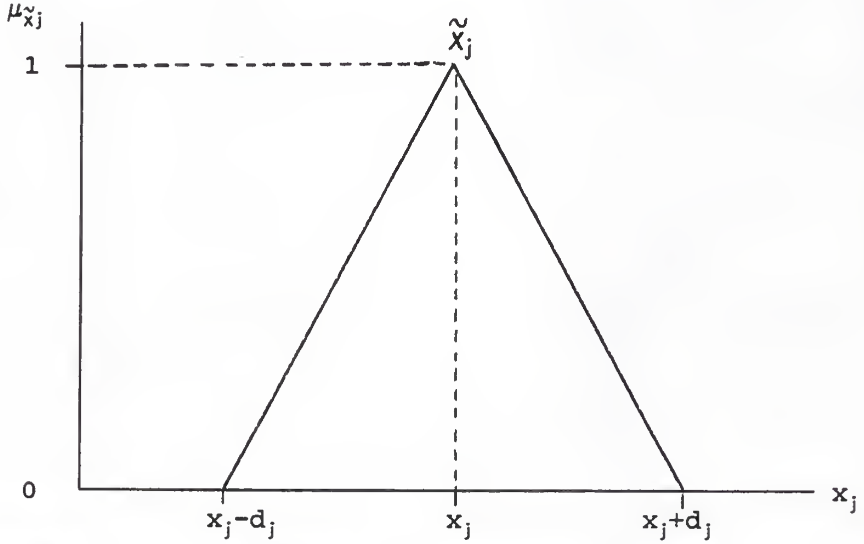


Figure 2.18. The membership function of the fuzzy decision variable $\tilde{x}_j$

$$
\begin{array}{ll}
\text{find} & x \\
\text{such that} & \tilde{Y}_0 = -\tilde{B}_0 + A_{01}\tilde{x}_1 + \dots + A_{0n}\tilde{x}_n \gtrsim 0 \\
& \tilde{Y}_1 = \tilde{B}_1 - (A_{11}\tilde{x}_1 + \dots + A_{1n}\tilde{x}_n) \gtrsim 0 \\
& \vdots \\
& \tilde{Y}_m = \tilde{B}_1 - (A_{11}\tilde{x}_1 + \dots + A_{1n}\tilde{x}_n) \gtrsim 0
\end{array}
\tag{2.35}
$$

Where $\tilde{B}_0 = (b_0, p_0)$, $\tilde{B}_i = (b_i, p_i)$, $A_{0j} = c_j$ and A_{ij}, for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$, are given. $\tilde{x}_j = (x_j, d_j)$, $\forall\, j$, are assumed.

Obviously, the problem (2.35) is similar to the problem given by (2.25), so we can solve it by a similar approach. However, it should be noted that the solution is a fuzzy solution with triangular membership function, and so the fuzzy objective value is Z^*.

Problem 4.

$$
\begin{array}{lll}
\text{maximize} & Z = c^T x & \\
\text{subject to} & (Ax)_i \leq \tilde{b}_i, & i = 1, 2, \dots, m \\
& x \geq 0. &
\end{array}
\tag{2.36}
$$

There are three approaches proposed by Tanaka, Okuda and Asai (1974), Werners (1984), and Verdegay (1982).

(i) α-level set approach -- Tanaka et al. [85]

Tanaka et al. proposed that the optimal solution (α^*, x^*) of the problem given by (2.36) will exist at

$$\alpha = \max_{x \in X_\alpha} f(x) \tag{2.37}$$

where X_α is α-level set: $X_\alpha = \{x \mid \mu_i \geq \alpha, \forall\ i, x \geq 0\}$, μ_i , $\forall$ i, are strongly fuzzy-convex}, and f(x) is defined as

$$f(x) = \frac{c^T x}{\max\limits_{x \in X_0} c^T x} \tag{2.38}$$

The problem given by (2.37) is then solved by the following algorithm:

<u>**Algorithm.**</u>

Step 1. Given α_1 for k = 1 and δ (stop criterion).

Step 2. Compute $f_k = \max\limits_{x \in X_\alpha} f(x)$

Step 3. Compute $\delta_k = \alpha_k - f_k$. If $|\delta_k| > \delta$, then go to Step 4. Otherwise, go to Step 5.

Step 4. Let $\alpha_{k+1} = \alpha_k - r_k \cdot (\delta_k)$. Then, go to Step 2, replacing k by k + 1. r_k is selected so that $r_k \geq 0$ and $0 \leq \alpha_{k+1} \leq 1$.

Step 5. Let $\alpha^* = \alpha_k$ and decide the optimal solution x^* such that $f(x^*) = \max\limits_{x \in X_{\alpha^*}} f(x)$

(ii) An Equivalent Symmetric Model -- Werners [131]

For the problem given by (2.36), the fuzzy resources may be determined as the intervals $[b_i, b_i + p_i]$, $\forall$ i. Werners defined Z^0 and Z^1 as follows:

$$
\begin{aligned}
Z^0 = \text{maximize} \quad & c^T x \\
\text{subject to} \quad & (Ax)_i \le b_i, \quad \forall\, i, \\
& x \ge 0, \\
Z^1 = \text{maximize} \quad & c^T x \\
\text{subject to} \quad & (Ax)_i \le b_i + p_i, \quad \forall\, i, \\
& x \ge 0.
\end{aligned}
\tag{2.39}
$$

Thus, we can construct a continuously nondecreasing linear membership function for the objective by use of Z^0 and Z^1, since the optimal value will exist in between Z^0 and Z^1, and the satisfaction of the optimal value will increase when its value increases. The membership function μ_0 (see Figure 2.19) of the objective is

$$
\mu_0 = \begin{cases} 1 & \text{if } c^T x > Z^1 \\ 1 - \dfrac{Z^1 - c^T x}{Z^1 - Z^0} & \text{if } Z^0 \le c^T x \le Z^1 \\ 0 & \text{if } c^T x < Z^0. \end{cases}
\tag{2.40}
$$

Therefore, by use of Zimmermann's symmetric model, the problem given by (2.36) can be solved by the following linear programming:

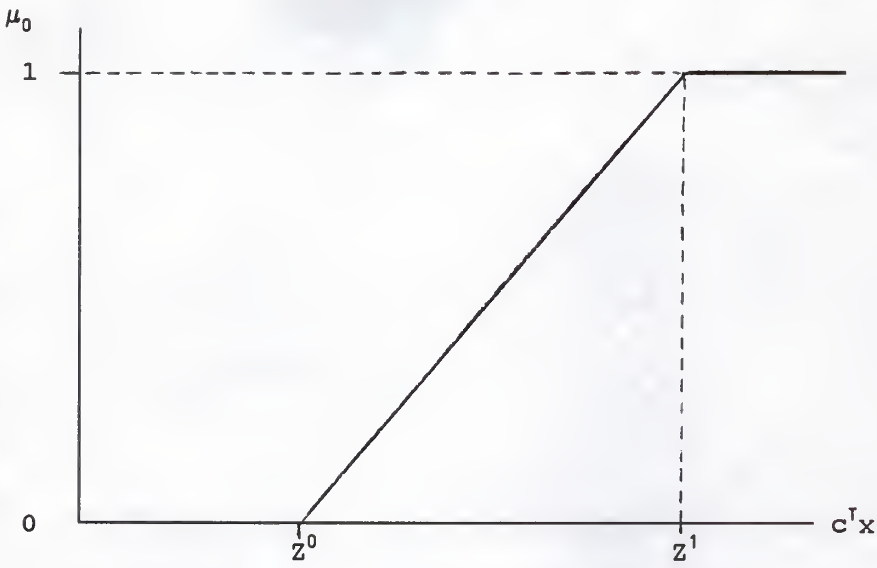


Figure 2.19. The membership function of the objective function

$$
\begin{aligned}
\text{maximize} \quad & \lambda \\
\text{subject to} \quad & \mu_0 \geq \lambda \\
& \mu_i \geq \lambda, \quad \forall \ i \\
& \lambda \in [0, 1]. \qquad (2.41)
\end{aligned}
$$

(iii) APrarmetric Approach -- Verdegay [96]

For the problem given by (2.36), Verdegay considered that if the membership functions of the fuzzy constraints:

$$
\mu_0 = \begin{cases} 1 & \text{if } (Ax)_i < b_i \\ 1 - \dfrac{(Ax)_i - b_i}{p_i} & \text{if } b_i \leq (Ax)_i \leq b_i + p_i \\ 0 & \text{if } (Ax)_i > b_i + p_i, \ \forall \ i. \end{cases} \qquad (2.42)
$$

are continuous and monotonic functions, then (2.36) will be equivalent to

$$\begin{aligned} &\text{maximize} && c^T x \\ &\text{subject to} && x \in X_\alpha, \end{aligned} \tag{2.43}$$

where $X_\alpha = \{x \mid \mu_i \geq \alpha,\ \forall\ i,\ x \geq 0\}$, for each $\alpha \in [0, 1]$.

or

$$\begin{aligned} &\text{maximize} && c^T x \\ &\text{subject to} && (Ax)_i \leq b_i + (1 - \alpha) p_i, \quad \forall\ i \\ & && x \geq 0 \text{ and } \alpha \in [0, 1], \end{aligned} \tag{2.44}$$

which is equivalent to a parametric programming as (2.17) while $\alpha = 1 - \theta$. Therefore, the fuzzy linear programming problem given (2.36) can be equivalent to a traditional parametric linear programming problem, when some proper forms of membership functions of the fuzzy constraints are assumed. It is noted that for each α, we have an optimal solution, so the solution with α grade of membership is actually fuzzy.

Problem 5.

$$\begin{aligned} &\widetilde{\text{maximize}} && \tilde{c}^T x \\ &\text{subject to} && (Ax)_i \leq b_i, \quad \forall\ i \\ & && x \geq 0. \end{aligned} \tag{2.45}$$

For the problem given by (2.45), Verdegay [97] indicated that it is really a dual problem of the Problem 4. Therefore, we can solve its dual problem by use of the approaches proposed

in Problem 4. On the other hand, we may want to solve this problem directly. Verdegay proposed the following equivalent formula for it:

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & \Phi(c) \geq 1 - \alpha \\ & (Ax)_i \leq b_i, \quad \forall\ i \\ & \alpha \in [0, 1] \text{ and } x \geq 0, \end{array} \tag{2.46}$$

where $c = (c_1, c_2, \ldots, c_n)$, $\Phi(c) = \inf_j \Phi_j(c_j)$, and $\Phi(c)$ is the membership function of $(c^T x)$ and $\Phi_j(c_j)$, $\forall\ j$, are the membership functions of c_j. The problem given by (2.46) is equivalent to

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & \Phi_j(c_j) \geq 1 - \alpha, \quad \forall\ j \\ & (Ax)_i \leq b_i, \quad \forall\ i \\ & \alpha \in [0, 1] \text{ and } x \geq 0, \end{array} \tag{2.47}$$

Therefore, we have

$$\begin{array}{ll} \text{maximize} & \Sigma_j\ c_j x_j \\ \text{subject to} & c_j \geq \Phi_j^{-1}(1 - \alpha), \quad \forall\ j \\ & (Ax)_i \leq b_i, \quad \forall\ i \\ & \alpha \in [0, 1] \text{ and } x \geq 0 \end{array} \tag{2.48}$$

which is equivalent to

$$\begin{aligned} \text{maximize} \quad & \Sigma_j\ c_j x_j \\ \text{subject to} \quad & c_j = \Phi_j^{-1}(1 - \alpha), \qquad \forall\ j \\ & (Ax)_i \le b_i, \qquad \forall\ i \\ & \alpha \in [0,\ 1] \text{ and } x \ge 0 \end{aligned} \tag{2.49}$$

or

$$\begin{aligned} \text{maximize} \quad & \Sigma_j\ \Phi_j^{-1}(1 - \alpha) x_j \\ \text{subject to} \quad & (Ax)_i \le b_i, \qquad \forall\ i \\ & \alpha \in [0,\ 1] \text{ and } x \ge 0. \end{aligned} \tag{2.50}$$

Obviously, (2.50) is a parameteric programming, so we can obtain the solution of (2.45) by solving (2.50). The solution is fuzzy and for each $(1 - \alpha)$-level cut of the fuzzy objective set, the solution has an α grade of preference.

Problem 6.

$$\begin{aligned} \text{maximize} \quad & c^T x \\ \text{subject to} \quad & \tilde{A}x \underset{\sim}{\le} \tilde{b} \\ & x \ge 0, \end{aligned} \tag{2.51}$$

where $\tilde{A} = (m,\ n,\ \beta,\ \tau)$ and $\tilde{b} = (s,\ t,\ \mathcal{E},\ \delta)$ are L-R fuzzy numbers, and depicted in Figure 2.20.

Ramik and Rimanek [75] proposed the following equivalent programming:

$$\begin{aligned} \text{maximize} \quad & c^T x \\ \text{subject to} \quad & A_{i1}x_1 + A_{i2}x_2 + \ldots + A_{in}x_n \le b_i, \qquad \forall\ i \\ & x \ge 0, \end{aligned} \tag{2.52}$$

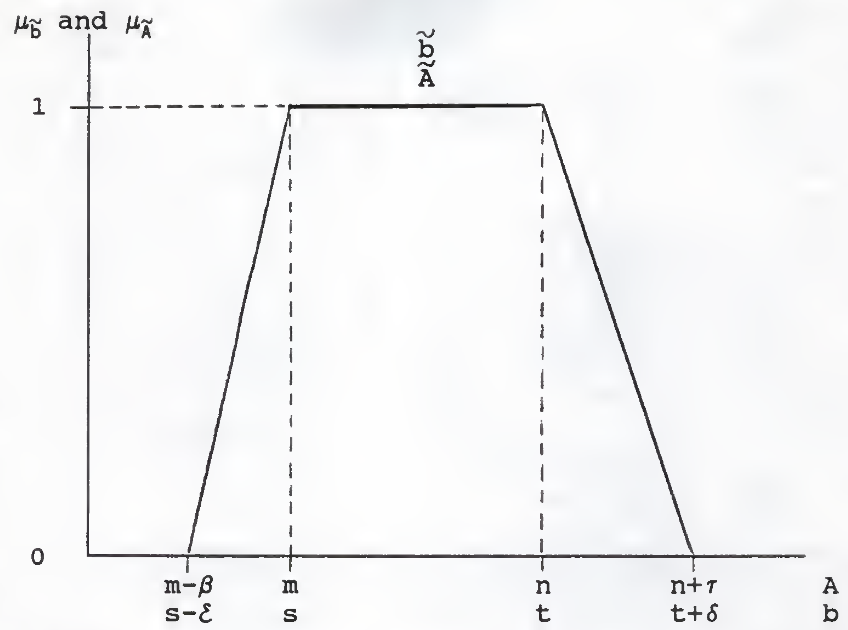


Figure 2.20. The membership functions of $\tilde{A}$ and $\tilde{b}$

where $A_{i1}x_1 \oplus A_{i2}x_2 \oplus \ldots \oplus A_{in}x_n = (\Sigma_j\ m_{ij}x_j,\ \Sigma_j\ n_{ij}x_j,\ \Sigma_j\ \beta_{ij}x_j, \Sigma_j\ \tau_{ij}x_j)$, $\forall$ i. The problem then may be equivalent to

$$\begin{array}{lll}
\text{maximize} & c^T x & \\
\text{subject to} & \Sigma_j\ m_{ij} \le s_i, & \forall\ i \\
& \Sigma_j\ (m_{ij} - \beta_{ij})x_j \le s_i - \mathcal{E}_i, & \forall\ i \\
& \Sigma_j\ n_{ij}x_j \le t_i, & \forall\ i \\
& \Sigma_j\ (n_{ij} + \tau_{ij})x_j \le t_i + \delta_i, & \forall\ i \\
& x \ge 0. & (2.53)
\end{array}$$

Now the problem of (2.53) is a traditional linear programming problem. The solution of it is then a crisp solution which is also the solution of Problem 6.

CHAPTER 3 INTERACTIVE FUZZY LINEAR PROGRAMMING

-- An Expert LP System

3.1 Introduction

After an overview of various models of fuzzy linear programming techniques, we now develop an interactive fuzzy linear programming approach which combines the advantages of Zimmermann's [123], Werners's [133], Chanas's [10] and Verdegay's [95] approaches and concepts. Each approach tackles one kind of linear programming problem-solving and decision making tasks. A systematic synthesis or aggregation of these approaches and beyond will provide an expert LP system for solving a specific domain of real-world LP problems.

In order to understand the differences and contributions of these approaches and concepts, it is necessary to discuss some major points among them in this section.

In the Zimmermann's fuzzy linear programming model, goal b_0 and its maximum tolerance p_0 should be given initially. In real-world problems, it is really a rather difficult job to initially ask the decision maker to give b_0 and p_0 without providing any information about them. Therefore, the membership function of the fuzzy objective is questionable and so the solution is also questionable. For example, if b_0 given is too large, there will be no solution in his FLP model. At the same time, if p_0 given is too large, then there

will be no meaning for the membership function. Thus, the solution is questionable. Instead of asking the decision maker b_0 and p_0 to establish the membership function of the fuzzy objective, Werners provided two possible extreme points Z^0 and Z^1 which are

$$Z^0 = \inf \left(\max_{x \in X} c^T x \right)$$

$$Z^1 = \sup \left(\max_{x \in X} c^T x \right)$$

where $X = \{x \mid (Ax)_i \le b_i, \forall\ i, \text{ and } x \ge 0\}$. The difference between Werners's and Zimmermann's membership function is depicted in Figure 3.1. In this figure, Zimmermann's b_0 and

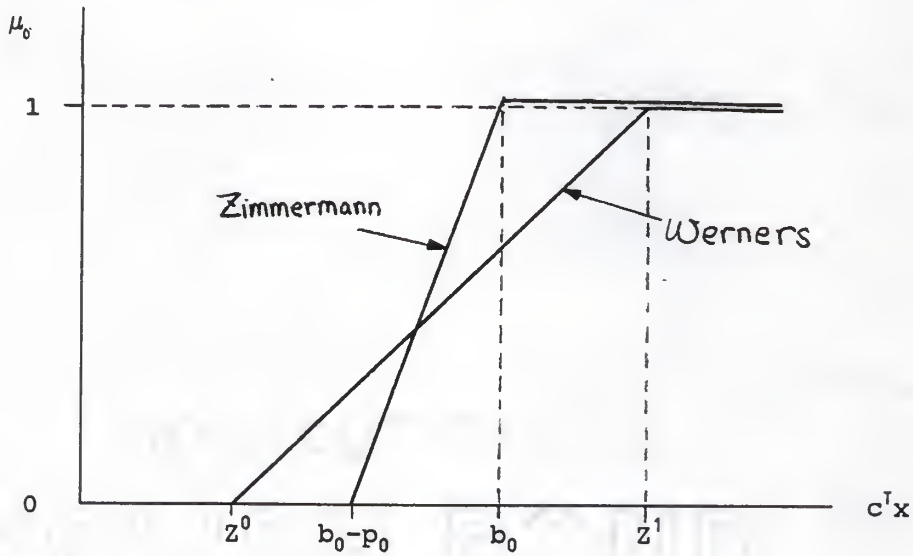


Figure 3.1. The difference between Zimmermann's and Werners's membership functions μ_0

p_0 are assumed to be rational, i.e., $Z^0 \leq b_0 \leq Z^1$ and $b_0 - p_0 \geq Z^0$. If so, the decision maker may consider Zimmermann's membership function as a more acceptable one some of the time.

While Zimmermann connected fuzzy sets theory and maxmin-operator proposed by Bellman and Zadeh (1970) with linear programming, Verdegay and Chanas obtained the equivalent relation between parametric programming with parametric resources and fuzzy linear programming with fuzzy resources defined by assummed linear membership functions. Thus, for a symmetric fuzzy linear programming problem, we can first consider its subproblem of nonsymmetric fuzzy linear programming with a crisp objective and fuzzy constraints, and then construct and solve its corresponding parametric programming problem. The solutions will be presented in a tableau which not only provide an optimal solution with respect to a parameter, but also provides the actually consumed resources. Therefore, the difficulty of providing b_0 and p_0 initially was overcome by presenting the solution table of a parametric programming problem. After referring to this solution table, the decision maker can precisely locate his subjective b_0 and p_0. The solution of Zimmermann's model is then reliable.

Chanas suggested that the membership function of the fuzzy constraints be constructed directly by the parametric optimal solution. Thus, μ_0 is a function of the parameter instead of the general function of x. However, in any real-

world problems, the number of constraints is always rather large (say 300), and so are the decision variables. Therefore, Chanas's approach for formulating the membership function of the fuzzy constraints is not practical.

The interactive fuzzy linear programming proposed here is a problem-oriented and user-dependent approach. It also considers a large variety of situations that the decision maker might meet, in modelling and solving a linear programming problem. The interactive fuzzy linear programming provides an efficient and systematic approach to solving the indicated linear programming problem, and then shows the results (solutions and resources used) to the decision maker. The decision maker may be satisfied with this solution or he may want to modify some situations and/or change the original model. If satisfied, the problem, of course, has been solved. If not satisfied, an interactive process will then proceed. The problem-solving procedure is continued until the decision maker obtains a satisfying solution. It is worth noting that the approach proposed here is not only to solve a given LP problem, but also is to design a high productivity system.

3.2 Interactive Fuzzy Linear Programming -- An Expert LP System

The approach proposed here is based on an interaction with the decision maker in order to find a satisfying solution for a linear programming problem. In a decision process using a LP model, resources available may be fuzzy, instead of a precisely given number as in the traditional LP model. For example, machine hours, labor force, material needed and so on, are always imprecise, because of a variety of potential suppliers. Therefore, they should be considered as fuzzy resources, and then the LP problem should be solved by use of the fuzzy sets theory. Thus under the fuzzy constraints, the objective function (e.g., profit function) to be maximized may be considered as either a crisp objective function or a fuzzy objective function symmetrically. Therefore, we can define a linear programming problem with crisp or fuzzy constraints, and a crisp or fuzzy objective as

$$
\begin{array}{lll}
\overset{(\sim)}{\text{maximize}} & Z = c^T x & \\
\text{subject to} & (Ax)_i \le \overset{(\sim)}{b_i}, & i = 1, 2, \ldots, m \\
 & x \ge 0, & \qquad\qquad (3.1)
\end{array}
$$

which is equivalent to the following problem, according to Goguen[43]:

$$
\begin{array}{lll}
\widetilde{\text{maximize}} & Z = c^T x & \\
\text{subject to} & (Ax)_i \underset{(\sim)}{\le} b_i, & i = 1, 2, \ldots, m \\
 & x \ge 0. &
\end{array}
$$

According to (3.1) problem, we can derive the following five problems:

Problem 1. When the resources can be determined precisely, a traditional linear programming problem is considered as

$$
\begin{aligned}
&\text{maximize} && Z = c^T x \\
&\text{subject to} && (Ax)_i \le b_i, \quad \forall\, i \\
& && x \ge 0, \qquad\qquad (3.2)
\end{aligned}
$$

where c, A, and b_i, $\forall$ i, are precisely given. The solution of (3.2) is a unique optimal solution.

Problem 2. [Approach of Chanas (1983) and Verdegay (1982)] A decision maker wishes to make a postoptimization analysis. Thus, a parametric programming problem is formulated as

$$
\begin{aligned}
&\text{maximize} && Z = c^T x \\
&\text{subject to} && (Ax)_i \le b_i + \theta p_i, \quad \forall\, i \\
& && \theta \in [0, 1] \text{ and } x \ge 0, \qquad\qquad (3.3)
\end{aligned}
$$

where c, A, b_i and p_i, $\forall$ i, are precisely given and θ is a parameter (a fraction of the maximum tolerances). p_i, $\forall$ i, are maximum tolerances which are always positive. The solutions $Z^*(\theta)$ of (3.3) problem are a function of θ. That is, for each θ, we can obtain an optimal solution; with the

Table 3.1. The solutions for a parametric programming problem

θ	$Z^*(\theta)$	resources actually used b_1	b_2	...	b_m
0.0					
0.1					
0.2					
.					
.					
.					
0.8					
0.9					
1.0					

use of resources increases, the optimal solution behavior can be figured out. The results are displayed in the Table 3.1 and then presented to the decision maker. The decision maker then can choose his satisfying solution for implementation.

On the other hand, the available resources may be fuzzy. Then the linear programming problem with fuzzy resources becomes

$$\begin{aligned} &\text{maximize} && Z = c^T x \\ &\text{subject to} && (Ax)_i \le \tilde{b}_i, \qquad \forall\ i \\ & && x \ge 0. \end{aligned} \tag{3.4}$$

It is possible to determine the maximum tolerances p_i, of the fuzzy resources $\tilde{b}_i$, $\forall$ i. Then we can construct the membership functions μ_i assumed linear for each fuzzy constraints, as follows:

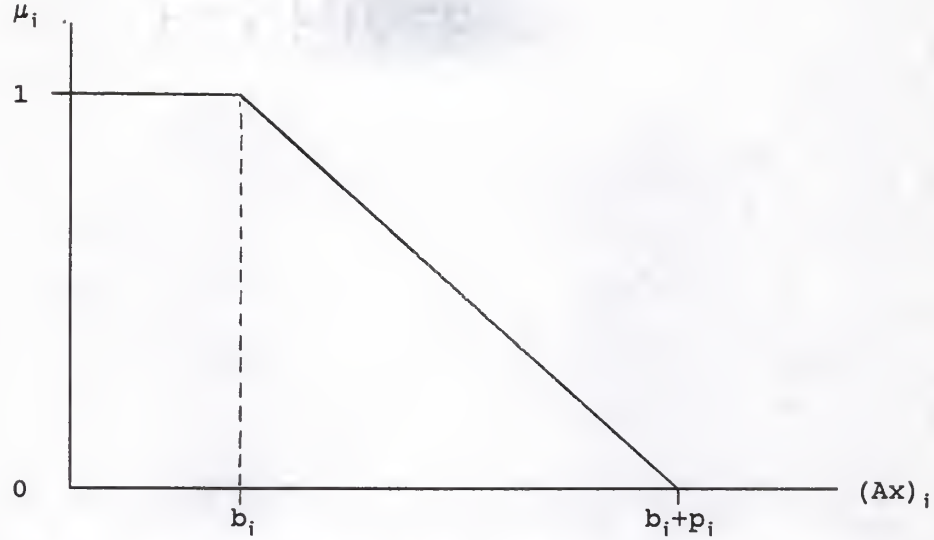


Figure 3.2. The membership functions of the fuzzy constraints

$$\mu_i = \begin{cases} 1 & \text{if } (Ax)_i < b_i \\ 1 - \dfrac{(Ax)_i - b_i}{p_i} & \text{if } b_i \le (Ax)_i \le b_i + p_i \\ 0 & \text{if } (Ax)_i > b_i + p_i \end{cases}$$

(see Figure 3.2). (3.5)

Verdegay (1982) and Chanas (1983) proposed that the problem of (3.4) and (3.5), however, is equivalent to equation (3.3), a parametric linear programming where c, A, b_i and p_i, $\forall$ i, are given, by use of the λ-level cut concept.

For each λ-level cut of the fuzzy constraint set, the problem of (3.4) becomes a traditional linear programming problem. That is,

$$\begin{aligned} &\text{maximize} && Z = c^T x \\ &\text{subject to} && x \in X_\lambda \end{aligned} \tag{3.6}$$

where $X_\lambda = \{x \mid \mu_i \geq \lambda\ ,\ \forall\ i,\ \text{and}\ x \geq 0,\ \lambda \in [0,\ 1]\}$. The problem of (3.6) is then equivalent to

$$\begin{aligned} &\text{maximize} && Z = c^T x \\ &\text{subject to} && (Ax)_i \leq b_i + (1 - \lambda) p_i, \quad \forall\ i \\ & && \lambda \in [0,\ 1] \text{ and } x \geq 0, \end{aligned} \tag{3.7}$$

where c, A, b_i and p_i, $\forall$ i, are precisely given. Now, if we set $\lambda = 1 - \theta$, the equation given by (3.7) will be the same as equation (3.3). Then a solution table is presented to the decision maker to determine the satisfying solution.

__Problem 3.__ [Werners's (1984) approach] A decision maker may want to solve a fuzzy linear programming problem with a fuzzy objective and fuzzy constraints, while the goal b_0 is not given. That is:

$$\begin{aligned} &\widetilde{\text{maximize}} && Z = c^T x \\ &\text{subject to} && (Ax)_i \leq \tilde{b}_i, \quad \forall\ i \\ & && x \geq 0, \end{aligned} \tag{3.8}$$

which is equivalent to

$$\begin{aligned} &\widetilde{\text{maximize}} && Z = c^T x \\ &\text{subject to} && (Ax)_i \leq b_i + \theta p_i, \quad \forall\ i\ , \\ & && \theta \in [0,\ 10] \text{ and } x \geq 0, \end{aligned} \tag{3.9}$$

where c, A, b_i and p_i, $\forall$ i, are given, but the goal of the fuzzy objective is not given.

To solve (3.9) by use of Werners's approach, let us first define Z^0 and Z^1 as follows:

$$Z^0 = \inf_{x \in X} (\max c^T x) = Z^*(\theta = 0), \tag{3.10}$$

$$Z^1 = \sup_{x \in X} (\max c^T x) = Z^*(\theta = 1), \tag{3.11}$$

where $X = \{x \mid (Ax)_i \leq b_i + \theta p_i,\ \forall\ i,\ \theta \in [0, 1] \text{ and } x \geq 0\}$,

or

$$\begin{aligned} Z^0 = \text{maximize} \quad & c^T x \\ \text{subject to} \quad & (Ax)_i \leq b_i, \quad \forall\ i \\ & x \geq 0, \end{aligned}$$

$$\begin{aligned} Z^1 = \text{maximize} \quad & c^T x \\ \text{subject to} \quad & (Ax)_i \leq b_i + p_i, \quad \forall\ i \\ & x \geq 0, \end{aligned}$$

Then, we can obtain Werners's membership function μ_0 of the fuzzy objective. That is:

$$\mu_0 = \begin{cases} 1 & \text{if } c^T x > Z^1 \\ 1 - \dfrac{Z^1 - c^T x}{Z^1 - Z^0} & \text{if } Z^0 \leq c^T x \leq Z^1 \\ 0 & \text{if } c^T x < Z^0, \end{cases} \tag{3.12}$$

(see Figure 3.3)

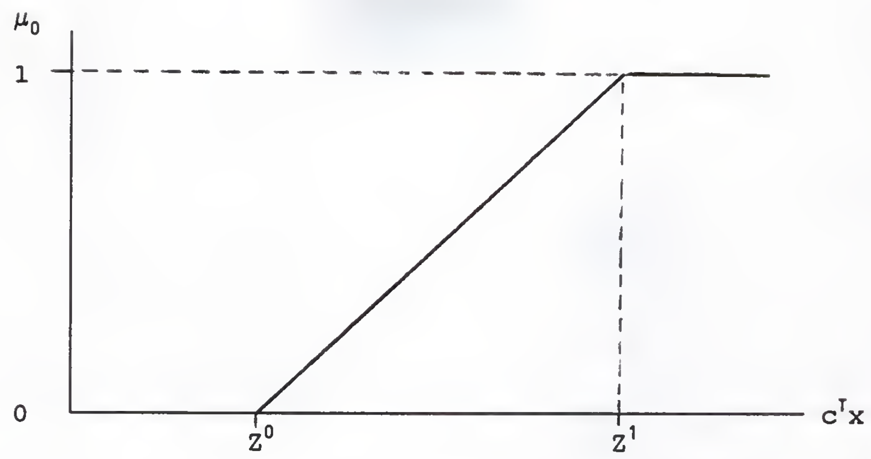


Figure 3.3. Werners's membership function of the fuzzy objective

The membership functions μ_i, $\forall$ i, of the fuzzy constraints are defined as (3.5).

By use of min-operator proposed by Bellman and Zadeh (1970), we can obtain the decision space D which is defined by its membership function μ_D where

$$\mu_D = \min\ (\mu_0, \mu_1, \ldots, \mu_m). \tag{3.13}$$

It is reasonable to choose the decision where μ_D is maximal as the optimal solution of the problem given by (3.8). Therefore, (3.8) is equivalent to

$$\text{maximize} \quad \lambda$$

$$\text{subject to} \quad \mu_0 = 1 - \frac{Z^1 - c^T x}{Z^1 - Z^0} \geq \lambda$$

$$\mu_i = 1 - \frac{(Ax)_i - b_i}{p_i} \geq \lambda, \quad \forall\ i$$

$$\lambda,\ \mu_0 \text{ and } \mu_i,\ \forall\ i,\ \epsilon\ [0, 1]$$

$$x \geq 0, \qquad (3.14)$$

where c, A, b_i and p_i, $\forall$ i, are given, and $\lambda = \mu_D = \min(\mu_0, \mu_1, \ldots, \mu_m)$. Let $\lambda = 1 - \theta$, then the problem given by (3.14) will be equivalent to

$$\text{minimize} \quad \theta$$

$$\text{subject to} \quad c^T x \geq Z^1 - \theta(Z^1 - Z^0)$$

$$(Ax)_i \leq b_i + \theta p_i, \quad \forall\ i$$

$$\theta\ \epsilon\ [0, 1] \text{ and } x \geq 0 \qquad (3.15)$$

where c, A, b_i and p_i, $\forall$ i, are given and θ is a fraction of $(Z^1 - Z^0)$ for the first constraint and a fraction of the maximum tolerances for others. The solution is a unique optimal solution.

Problem 4. [Zimmerman's (1976) Approach] A decision maker may want to solve a fuzzy linear programming problem with a fuzzy objective and fuzzy constraints, when the goal b_0 of the fuzzy objective and its maximum tolerence are given. That

is:

$$\begin{array}{lll} \widetilde{\text{maximize}} & c^T x & \\ \text{subject to} & (Ax)_i \lesssim b_i, & \forall\ i \\ & x \geq 0, & \end{array} \qquad (3.16)$$

where c, A, b_0, p_0, b_i and p_i, $\forall$ i, are given. It is noted that the decision maker should refer to Table 3.1 for deciding b_0 and p_0. The problem given by (3.16) is actually equivalent to

$$\begin{array}{lll} \text{find} & x & \\ \text{such that} & c^T x \gtrsim b_0 & \\ & (Ax)_i \lesssim b_i, & \forall\ i \\ & x \geq 0, & \end{array} \qquad (3.17)$$

with the membership functions of the fuzzy constraints as previously described in (3.5) and the membership function of the fuzzy objective μ_0 as follows:

$$\mu_0 = \begin{cases} 1 & \text{if } c^T x > b^0 \\ 1 - \dfrac{b_0 - c^T x}{p_0} & \text{if } b_0 - p_0 \leq c^T x \leq b_0 \\ 0 & \text{if } c^T x < b_0 - p_0 \end{cases} \qquad (3.18)$$

(see Figure 3.4).

Thus, by use of the maxmin concept, the problem (3.7) is actually equivalent to

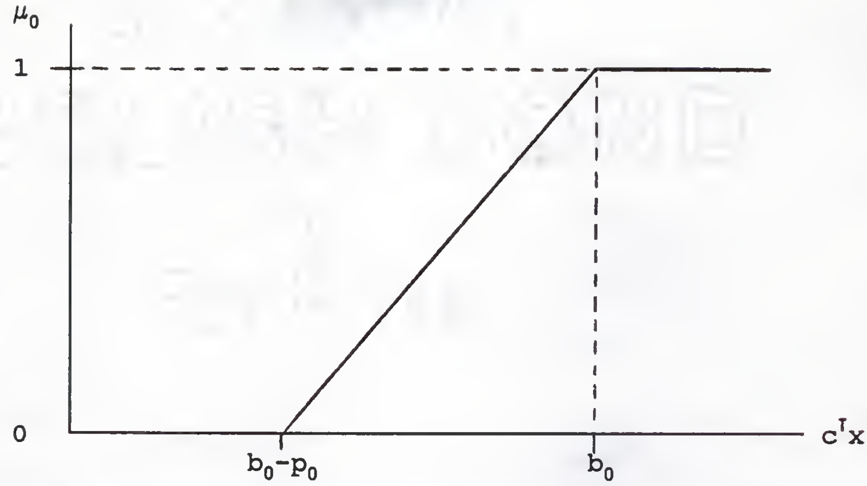


Figure 3.4. Zimmermann's membership function of the fuzzy objective

$$\begin{aligned}
&\text{maximize} && \lambda \\
&\text{subject to} && \mu_0 = 1 - \frac{b_0 - c^Tx}{p_0} \geq \lambda \\
& && \mu_i = 1 - \frac{(Ax)_i - b_i}{p_i} \geq \lambda, \quad \forall\, i \\
& && \lambda,\ \mu_0 \text{ and } \mu_i,\ \forall\, i,\ \epsilon\ [0, 1] \\
& && x \geq 0, \qquad\qquad (3.19)
\end{aligned}$$

where c, A, b_0, p_0, b_i and p_i, $\forall$ i, are given. Let $\lambda = 1 - \theta$, then the problem (3.14) will be equivalent to

$$\begin{aligned}
&\text{minimize} && \theta \\
&\text{subject to} && c^Tx \geq b_0 - \theta p_0 \\
& && (Ax)_i \leq b_i + \theta p_i, \quad \forall\, i \\
& && \theta\ \epsilon\ [0, 1] \text{ and } x \geq 0 \qquad\qquad (3.20)
\end{aligned}$$

where c, A, b_0, p_0, b_i and p_i, $\forall$ i, are given and θ is a fraction of the maximum tolerances. The optimal solution of the problem (3.20) is unique.

Problem 5. A decision maker may want to solve a fuzzy linear programming problem with a fuzzy objective and fuzzy constraints, while only the goal b_0 of the fuzzy objective is given, but its tolerance p_0 is not given. That is:

$$\begin{array}{lll} \text{maximize} & c^T x & \\ \text{subject to} & (Ax)_i \leq b_i, & \forall i \\ & x \geq 0, & \end{array} \qquad (3.21)$$

where a, A, b_0, b_i and p_i, $\forall$ i, are given, but p_0 is not given. While p_0 is not given. We do know that p_0 should be in between 0 and $b_0 - Z^0$ (Z^0 as defined in the problem 4 section). For each $p_0 \in [0, b_0 - Z^0]$, we can obtain the membership function of the fuzzy objective as (3.18). Since in a high-productivity system the objective value should be larger then Z^0 at $\theta = 0$, there is no meaning to giving a positive grade of membership for those which are less than Z^0. Figure 3.5 depicts the possible range of p_0.

The difference between Problem 5 and Problem 4 is that p_0 is not initially given in Problem 5. Therefore, we may assume a set of p_0s, where $p_0 \in [0, b_0 - Z^0]$. Then, the problem of each p_0 given is a Problem 4. The solutions for

this given set of p_0s are presented in Table 3.2. After referring to this table, the decision maker may choose a refined p_0. Then a Problem 4 with the decision maker's refined p_0 is solved. This solution will be the final optimal solution for the problem (3.21).

Now it is time to discuss our interactive fuzzy linear programming -- an expert LP system in the following algorithm (see Figure 3.6):

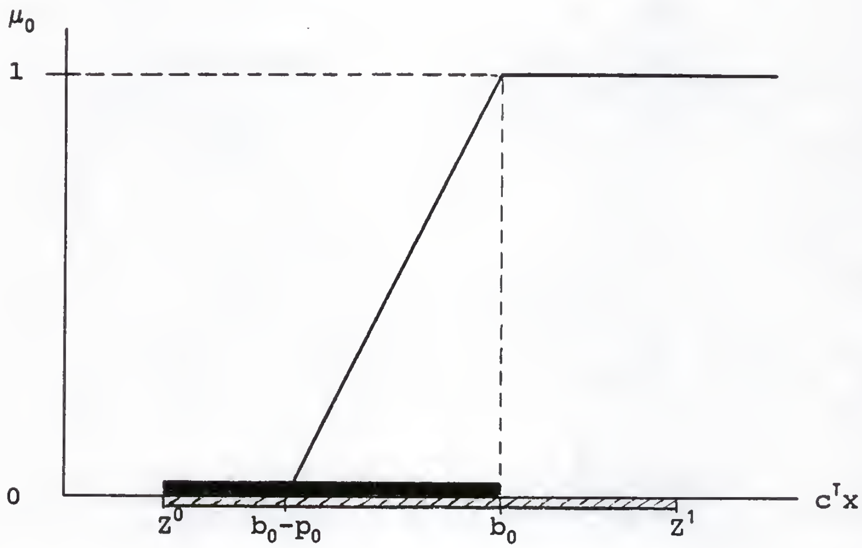


Figure 3.5. The reasonable range of p_0

Table 3.2. The optimal solution of a symmetric fuzzy linear programming for a given set of p_0s

				resources actually used			
p_0	θ	Z^{**}	x^{**}	b_1	b_2	. . .	b_m
0							
.							
.							
.							
b_0-Z^0							

Algorithm.

Step 1. Solve a traditional linear programming problem of equation (3.2) by use of the simplex method. The unique optimal solution with its corresponding consumed resources is presented to the decision maker.

Step 2. Does this solution satisfy the decision maker? If not, the decision maker may want to reduce the resources available b_i, for some i, and then go back to Step 1 until inventory stays at a minimal acceptable level while the optimal profit is obtained. Thus, he can reach a high-productivity system design. On the other hand, the decision maker may want to make a parametric analysis of the resources available, or consider the resources as fuzzy resources because of a variety of suppliers. Then, go to Step 3. Of course, the decision maker might be satisfied with this solution, and then terminate the solution procedure.

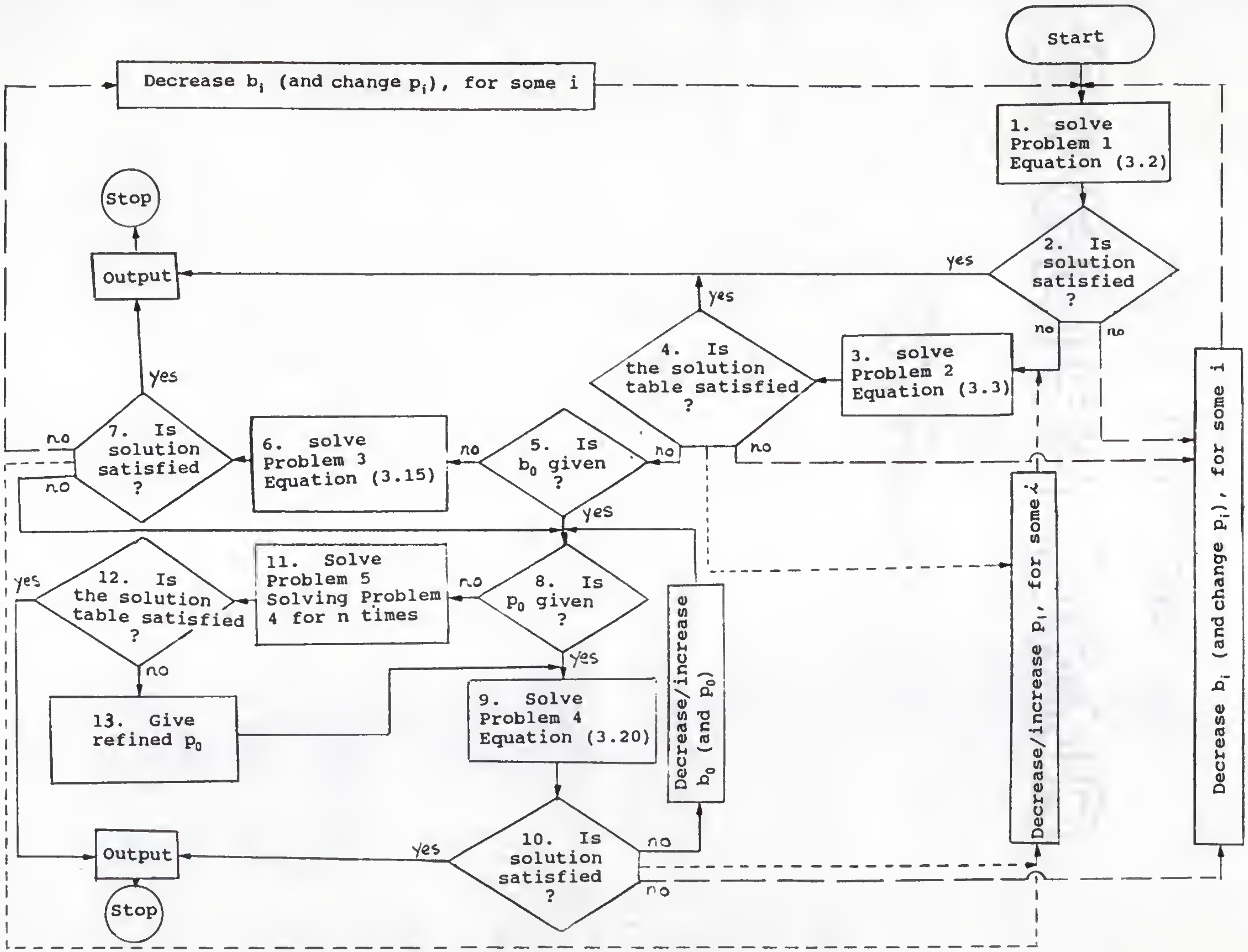


Figure 3.6. The flow chart of interactive fuzzy linear programming -- an expert LP system

Step 3. Solve a parametric linear programming problem of the equation (3.3). The results are depicted in Table 3.1. At the same time, let us identify $Z^0 = Z^*(\theta = 0)$ and $Z^1 = Z^*(\theta = 1)$.

Step 4. Does any of these solutions shown in Table 3.1 satisfy the decision maker? If yes, print out the solution and then terminate the solution procedure. If not, the following procedures may be considered:

(i) decrease b_i (and change p_i) for some i, and then go to Step 1.

(ii) increase/decrease p_i for some i, and then go to Step 3.

(iii) consider a symmetric fuzzy linear programming approach with a fuzzy objective and fuzzy constraints, and then go to Step 5.

Step 5. After referring to Table 3.1, the decision maker is then asked for his subjective goal b_0 and its tolerance p_0 for solving a symmetric fuzzy linear programming problem. If the decision maker does not like to give his goal for the fuzzy objective, go to Step 6. If b_0 is given, go to Step 8.

Step 6. Solve Problem 3 of equation (3.15). A unique Werners's solution is then provided.

Step7. Is the solution of Problem 3 satisfying? If yes, print out the solution and then terminate the solution procedure. Otherwise, the following steps may be considered:

(i) determine the goal b_0, and then go to Step 8.

(ii) decrease b_i (and change p_i) for some i, and then go to Step 1.

(iii) increase/decrease p_i for some i, and then go to Step 3.

Step 8. Is p_0 determined by the decision maker? If the decision maker would like to specify p_0, we should provide Table 3.1 to help the decision maker. It is noted that $p_0 \in [0, b_0 - Z^0]$. Then go to Step 9. If p_0 is not given, then go to Step 11.

Step 9. Solve Problem 4 of equation (3.20). A unique Zimmermann's solution is obtained.

Step 10. Is the solution of Problem 4 satisfying? If yes, print out the solution and then terminate the solution procedure. If not, the following steps are considered:

(i) decrease b_i (and change p_i) for some i, and then go to Step 1.

(ii) increase/decrease p_i for some i, and then go to Step 3.

(iii) decrease/increase b_0 (and p_0), and then go to Step 8.

Step 11. Solve Problem 5. That is: call Step 9 to solve Problem 4 for a set of p_0s. Of course, $p_0 \in [0, b_0 - Z^0]$. Then the solutions are depicted in Table 3.2.

Step 12. Are the solutions satisfying? If yes, print out the solution and then terminate the solution procedure. Otherwise, ask the decision maker to specify the refined p_0,

and then go to Step 9. It is rather reasonable to ask the decision maker p_0 at this step, because he has a good idea about p_0 now.

Note.

For implementing the above interactive fuzzy linear programming, we need only two solution-finding techniques: simplex method and parametric method. Therefore, the interactive fuzzy linear programming approach proposed here can be easily programmed in a PC system for its simplicity.

3.3 Example -- A Product - Mix Selection Problem [98]

Suppose that the Knox Mix company has the option of using one or more of four different types of production processes. The first and second processes yield items of product A, and the third and fourth yield items of product B. The inputs for each process are labor measured in man-weeks. pounds of material Y, and boxes of material Z. Since each process varies in its input requirements, the profitabilities of the process differ, even for processes producing the same item. The manufacturer, deciding on a week's production schedule, is limited in the range of possibilities by the available amounts of manpower and of both kinds of raw materials. The full technology and input restrictions are given in the following table:

Table 3.3. The input data for the Knox product-mix selection problem

Item	Man-weeks	Material-Y (pounds)	Material-Z (boxes)	Unit profit
One item of prodect A				
process 1	1	7	3	4
process 2	1	5	5	5
One item of prodect B				
process 3	1	3	10	9
process 4	1	2	15	11
Total available	≤ 15	≤ 120	≤ 100	maximize

Suppose that the production levels in processes 1, 2, 3, and 4 are x_1, x_2, x_3 and x_4, respectively. The problem then can be formulated as the following linear programming problem:

$$
\begin{array}{lll}
\max & 4x_1 + 5x_2 + 9x_3 + 11x_4 & \text{(profit)} \\
\text{s.t.} & x_1 + x_2 + x_3 + x_4 \le 15 & \text{(man-weeks)} \\
& 7x_1 + 5x_2 + 3x_3 + 2x_4 \le 120 & \text{(material Y)} \\
& 3x_1 + 5x_2 + 10x_3 + 15x_4 \le 100 & \text{(material Z)} \\
& x_1,\ x_2,\ x_3 \text{ and } x_4 \ge 0. &
\end{array}
$$

Now, let us start our interactive fuzzy linear programming system to obtain a satisfying solution.

Step 1. Solve the above linear programming problem by use of the simplex method. The final tableau is shown in Table 3.4.

Table 3.4. The final tableau of the simplex method for the Knox product-mix selection problem

Basic variable		4 x_1	5 x_2	9 x_3	11 x_4	0 s_1	0 s_2	0 s_3	RHS
4	x_1	1	5/7	0	-5/7	10/7	0	-1/7	50/7
0	s_2	0	-6/7	0	13/7	-61/7	1	4/7	325/7
9	x_3	0	2/7	1	12/7	-3/7	0	1/7	55/7
$z_j - c_j$		0	3/7	0	11/7	13/7	0	5/7	695/7

The optimal solution is: $x^* = (50/7, 0, 55/7, 0) = (7.14, 0, 7.86, 0)$ and $Z^* = \$695/7 = \$99.29$. The actual used resources are 15, 73.57 and 100 units for man-weeks, material Y and material Z, repectively.

Step 2. The decision maker feels that 73.57 units of material

actually is enough to reach $99.25 profit. Thus, 120 - 73.57 = 46.43 pounds of material Y are really idle resources. The decision maker, therefore, would like to try to find the solution of a new linear programming with the resources changed from $(b_1, b_2, b_3) = (15, 120, 100)$ to $(15, 73.57, 100)$. Then, go to Step 1.

Step 1. Solve the following linear programming problem:

$$\begin{aligned} \text{maximize} \quad & 4x_1 + 5x_2 + 9x_3 + 11x_4 \\ \text{subject to} \quad & x_1 + x_2 + x_3 + x_4 \le 15 \\ & 7x_1 + 5x_2 + 3x_3 + 2x_4 \le 73.57 \\ & 3x_1 + 5x_2 + 10x_3 + 15x_4 \le 100 \\ & x_1,\ x_2,\ x_3 \text{ and } x_4 \ge 0. \end{aligned}$$

The optimal solution is the same as previously indicated: $x^* = (7.14, 0, 7.86, 0)$, and $Z^* = \$99.29$. The actual used resources are 15, 73.57, and 100. Now there are no idle resources. Instead of just optimizing a given L.P. system, the procedure has yielded a high-productivity system.

Step 2. If the decision maker is satisfied with this solution, print out the results and then stop the solution procedure. Sometimes, a decision maker may want to further analyze his problem. Then the following two cases may be considered.

Case 1.

Step 3-1. The decision maker may feels that a posterior analysis of the previous LP problem is necessary before implementing its solution.

Let us assume that the decision maker provides that the maximum tolerances for man-weeks, material Y and material Z are 3, 0, and 20 units, respectively. Then, the parametric programming is

$$\begin{aligned} \text{maximize} \quad & 4x_1 + 5x_2 + 9x_3 + 11x_4 \\ \text{subject to} \quad & x_1 + x_2 + x_3 + x_4 \le 15 + 3\theta \\ & 7x_1 + 5x_2 + 3x_3 + 2x_4 \le 120 \\ & 3x_1 + 5x_2 + 10x_3 + 15x_4 \le 100 + 20\theta \\ & x_1,\ x_2,\ x_3 \text{ and } x_4 \ge 0. \end{aligned}$$

where θ is a parameter. By use of the parametric technique and the final table of the simplex method shown in Table 3.4, we can obtain the following results:

$$(10/7,\ 0,\ -1/7) \begin{pmatrix} 3\theta \\ 0 \\ 20\theta \end{pmatrix} = 10\theta/7$$

$$(-61/7,\ 1,\ 4/7) \begin{pmatrix} 3\theta \\ 0 \\ 20\theta \end{pmatrix} = -103\theta/7$$

$$(-3/7,\ 0,\ 1/7) \begin{pmatrix} 3\theta \\ 0 \\ 20\theta \end{pmatrix} = 11\theta/7$$

The final simplex table is shown in Table 3.5. Since the RHS $50/7 + 10\theta/7$, $325/7 - 103\theta/7$ and $55/7 + 11\theta/7$, for $\theta \in [0, 1]$,

are always greater than zero, the optimal solution is then: $x^* = (7.14 + 1.43\theta, 0, 7.86 + 1.57\theta, 0)$, and $Z^* = \$(99.29 + 19.86\theta)$. In order to more clearly display these results, Table 3.6 is provided to the decision maker.

Table 3.5. The final parametric tableau of the simplex method for the Knox product-mix selection problem

Basic variable		4 x_1	5 x_2	9 x_3	11 x_4	0 s_1	0 s_2	0 s_3	
4	x_1	1	5/7	0	-5/7	10/7	0	-1/7	$50/7+10\theta/7$
0	s_2	0	-6/7	0	13/7	-61/7	1	4/7	$325/7+103\theta/7$
9	x_3	0	2/7	1	12/7	-3/7	0	1/7	$55/7+11\theta/7$
$z_j - c_j$		0	3/7	0	11/7	13/7	0	5/7	$695/7+139\theta/7$

Table 3.6. The solutions of the parametric programming problem

θ	Z^*	resources actually used man-weeks	material Y	material Z
0.0	99.29	15.00	73.57	100.00
0.1	101.28	15.30	75.04	102.00
0.2	103.27	15.60	76.51	104.00
0.3	105.26	15.90	77.98	106.00
0.4	107.25	16.20	79.45	108.00
0.5	109.24	16.50	80.92	110.00
0.6	111.23	16.80	82.39	112.00
0.7	113.22	17.10	83.86	114.00
0.8	115.21	17.40	85.33	116.00
0.9	117.20	17.70	86.80	118.00
1.0	119.19	18.00	88.27	120.00

Step 4-1. After an overview of the above table, the decision maker may pick out a satisfying solution for implementation. If so, print out the results and terminate the solution procedure. Otherwise, the following steps may be considered:

(i) reduce the amount of material Y from 120 to 73.57 and give its maximum tolerance. Then go to Step 1.

(ii) decrease/increase the maximum tolerances p_1 and/or p_3, and then go to Step 3-1. (just modify the results of the parametric process)

The solution process will continue until the satisfying solution is reached.

Case 2.

Step 3-2. The decision maker may consider that the traditional linear programming is not enough to solve his problem because of the imprecise properties of the resources in nature. After detailed analysis, he feels that the consumption of man-weeks, material Y and material Z should be "substantially less than or equal to" 15, 80 and 100 units, respectively, with the corresponding maximum tolerances of 5, 40 and 30 units. Thus the problem becomes a fuzzy linear programming problem with a crisp objective and fuzzy constraints which are defined by their membership function assumed to be linear. That is:

$$\begin{array}{ll} \text{maximize} & 4x_1 + 5x_2 + 9x_3 + 11x_4 \\ \text{subject to} & x_1 + x_2 + x_3 + x_4 \le 15 \\ & 7x_1 + 5x_2 + 3x_3 + 2x_4 \le 80 \\ & 3x_1 + 5x_2 + 10x_3 + 15x_4 \le 100 \\ & x_1,\ x_2,\ x_3 \text{ and } x_4 \ge 0. \end{array}$$

where the membership function μ_i for the ith fuzzy constraints, $\forall$ i, are

$$\mu_1 = \begin{cases} 1 & \text{if } x_1+x_2+x_3+x_4 < 15 \\ 1 - \dfrac{x_1+x_2+x_3+x_4 - 15}{5} & \text{if } 15 \le x_1+x_2+x_3+x_4 \le 20 \\ 0 & \text{if } x_1+x_2+x_3+x_4 > 20 \end{cases}$$

$$\mu_1 = \begin{cases} 1 & \text{if } 7x_1+5x_2+3x_3+2x_4 < 80 \\ 1 - \dfrac{7x_1+5x_2+3x_3+2x_4 - 80}{40} & \text{if } 15 \le 7x_1+5x_2+3x_3+2x_4 \le 120 \\ 0 & \text{if } 7x_1+5x_2+3x_3+2x_4 > 120 \end{cases}$$

$$\mu_1 = \begin{cases} 1 & \text{if } 3x_1+5x_2+10x_3+15x_4 < 100 \\ 1 - \dfrac{3x_1+5x_2+10x_3+15x_4 - 100}{30} & \text{if } 100 \le 3x_1+5x_2+10x_3+15x_4 \le 130 \\ 0 & \text{if } 3x_1+5x_2+10x_3+15x_4 > 130 \end{cases}$$

This nonsymmetric fuzzy linear programming is then equivalent to

$$\begin{array}{ll} \text{maximize} & 4x_1 + 5x_2 + 9x_3 + 11x_4 \\ \text{subject to} & x_1 + x_2 + x_3 + x_4 \leq 15 + 5\theta \\ & 7x_1 + 5x_2 + 3x_3 + 2x_4 \leq 80 + 40\theta \\ & 3x_1 + 5x_2 + 10x_3 + 15x_4 \leq 100 + 30\theta \\ & x_1,\ x_2,\ x_3 \text{ and } x_4 \geq 0, \text{ and } \theta \in [0, 1]. \end{array}$$

where θ is a parameter. Therefore, we can use the parametric programming approach to solve this problem and obtain its solutions as shown in Table 3.7.

Table 3.7. The solutions of a nonsymmetric fuzzy linear programming problem with a crisp objective and fuzzy constraints

		resources actually used		
θ	Z^*	man-weeks	material Y	material Z
0.0	99.29	15.00	73.57	100.00
0.1	102.36	15.50	76.22	103.00
0.2	105.43	16.00	78.75	106.00
0.3	108.50	16.50	81.50	109.00
0.4	111.57	17.00	84.15	112.00
0.5	114.64	17.50	86.80	115.00
0.6	117.71	18.00	89.45	118.00
0.7	120.78	18.50	92.10	121.00
0.8	123.85	19.00	94.75	124.00
0.9	126.92	19.50	97.40	127.00
1.0	130.00	20.00	100.00	130.00

Step 4-2. After referring to this solution table, the decision maker may choose a satisfying solution for implementation, and then terminate the solution procedure.

Otherwise, the following procedures may be considered:

(i) decrease b_i (and change p_i) for some i, and then go to Step 1.

(ii) decrease/increase p_i for some i, and then go to Step 3.

(iii) consider a symmetric fuzzy linear programming with a fuzzy objective and fuzzy constraints, and then go to Step 5.

Assume that the situation (iii) is considered here.

Step 5. Present Table 3.7 to the decision maker and ask him to determine his subjective goal b_0. If b_0 is given, go to Step 8. Otherwise, go to the next step.

Step 6. Solve Problem 3 given in (3.15). First, according to the table 3.7, let us identify

$$Z^0 = Z_*(\theta = 0) = 99.29$$

$$Z^1 = Z^*(\theta = 1) = 130.$$

The symmetric linear programming problem then can be formulated as follows:

$$\begin{array}{ll} \widetilde{\text{maximize}} & 4x_1 + 5x_2 + 9x_3 + 11x_4 \\ \text{subject to} & x_1 + x_2 + x_3 + x_4 \lesssim 15 \\ & 7x_1 + 5x_2 + 3x_3 + 2x_4 \lesssim 80 \\ & 3x_1 + 5x_2 + 10x_3 + 15x_4 \lesssim 100 \\ & x_1,\ x_2,\ x_3 \text{ and } x_4 \geq 0. \end{array}$$

which is equivalent to

$$\begin{array}{ll} \text{find} & x \\ \text{such that} & 4x_1 + 5x_2 + 9x_3 + 11x_4 \gtrsim 130 \\ & x_1 + x_2 + x_3 + x_4 \lesssim 15 \\ & 7x_1 + 5x_2 + 3x_3 + 2x_4 \lesssim 80 \\ & 3x_1 + 5x_2 + 10x_3 + 15x_4 \lesssim 100 \\ & x_1,\ x_2,\ x_3 \text{ and } x_4 \geq 0. \end{array}$$

where the membership functions of the fuzzy constraints are described in Step 3-2 and the membership function μ_0 of the fuzzy objective is defined as

$$\mu_0 = \begin{cases} 1 & \text{if } 4x_1+5x_2+9x_3+11x_4 > 130 \\ 1 - \dfrac{130 - 3x_1+5x_2+10x_3+15x_4}{130 - 99.29} & \text{if } 99.29 \leq 4x_1+5x_2+9x_3+11x_4 \leq 130 \\ 0 & \text{if } 4x_1+5x_2+9x_3+11x_4 > 99.29 \end{cases}$$

This problem is actually equivalent to

$$\begin{array}{ll} \text{minimize} & \theta \\ \text{subject to} & 4x_1 + 5x_2 + 9x_3 + 11x_4 \geq 130 - 30.71\theta \\ & x_1 + x_2 + x_3 + x_4 \leq 15 + 5\theta \\ & 7x_1 + 5x_2 + 3x_3 + 2x_4 \leq 80 + 40\theta \\ & 3x_1 + 5x_2 + 10x_3 + 15x_4 \leq 100 + 30\theta \\ & x_1,\ x_2,\ x_3 \text{ and } x_4 \geq 0 \text{ and } \theta \in [0,\ 1]. \end{array}$$

The solution is: $x^{**} = (8.57,\ 0,\ 8.93,\ 0)$ and $Z^{**} = \$114.65$ at

$\theta = 0.5$, while the actually used resources are 17.5, 86.78 and 115.01 for man-weeks, material Y and material Z, respectively.

Step 7. The decision maker may satisfy this solution, and then print out the results and stop the solution procedure. If he is not satisfied with this solution, the following steps might be considered:

(i) decrease b_i (and change p_i) for some i, and then go to Step 1.

(ii) decrease/increase p_i for some i, and then go to Step 3.

(iii) determine b_0, after recalling Table 3.7, and then go to Step 8.

Let us assume that the situation (iii) is considered.

Step 8. With presenting Table 3.7, ask the decision maker to determine p_0. If p_0 is given, go to Step 9. Otherwise, go to Step 11.

Step 9. Solve Problem 4 given (3.20). Now let us assume $b_0 = 111.57$ at $\theta = 0.4$ and $p_0 = 10$. It is noted that p_0 should be in between 0 and 12.28. Our problem becomes:

$$
\begin{array}{ll}
\text{find} & x \\
\text{such that} & 4x_1 + 5x_2 + 9x_3 + 11x_4 \underset{\sim}{\geq} 111.57 \\
& x_1 + x_2 + x_3 + x_4 \underset{\sim}{\leq} 15 \\
& 7x_1 + 5x_2 + 3x_3 + 2x_4 \underset{\sim}{\leq} 80 \\
& 3x_1 + 5x_2 + 10x_3 + 15x_4 \underset{\sim}{\leq} 100 \\
& x_1,\ x_2,\ x_3 \text{ and } x_4 \geq 0.
\end{array}
$$

where the membership functions of the fuzzy constraints are described in Step 3-2 and the membership function μ_0 of the fuzzy objective is defined as

$$\mu_0 = \begin{cases} 1 & \text{if } 4x_1+5x_2+9x_3+11x_4 > 111.57 \\ 1 - \dfrac{111.57-3x_1+5x_2+10x_3+15x_4}{10} & \text{if } 101.57 \leq 4x_1+5x_2+9x_3+11x_4 \leq 111.57 \\ 0 & \text{if } 4x_1+5x_2+9x_3+11x_4 > 101.57 \end{cases}$$

The problem is then equivalent to

$$\begin{array}{lll} \text{minimize} & \theta & \\ \text{subject to} & 4x_1 + 5x_2 + 9x_3 + 11x_4 \geq 111.57 - 10\theta \\ & x_1 + x_2 + x_3 + x_4 \leq 15 + 5\theta \\ & 7x_1 + 5x_2 + 3x_3 + 2x_4 \leq 80 + 40\theta \\ & 3x_1 + 5x_2 + 10x_3 + 15x_4 \leq 100 + 30\theta \\ & x_1,\ x_2,\ x_3 \text{ and } x_4 \geq 0 \text{ and } \theta \in [0,\ 1]. \end{array}$$

The solution is: $x^{**} = (8.01, 0, 8.50, 0)$ and $Z^{**} = \$108.54$ at $\theta = 0.30$, while the actually used resources are 16.51, 81.57 and 109.03 for man-weeks, material Y and material Z, respectively.

Step 10. Considered the following steps:

(i) decrease b_i (and change p_i) for some i, and then go to Step 1.

(ii) decrease/increase p_i for some i, and then

go to Step 3.

(iii) change b_0 (and p_0), and then go to Step 8.

(iv) stop the solution procedure if the solution is satisfied.

Here, let us assume that the solution is satisfied. Then the solution procedure of our interactive fuzzy linear programming system is completed. After such detailed communication between the system and the decision maker, the solution will be quite good for implementation.

Step 11. Take a simulation by choosing a set of p_0s. Let us take 5 possible value $p_0 \in [0, 12.28]$; 0, 3, 6, 9, 12.28. For each p_0 value, we call Step 9 to obtain the solution of Problem 4. The results of this simulation are depicted in Table 3.8.

Table 3.8. The solutions of Problem 5

p_0	θ^*	Z^{**}	x^{**}
0	0.400	111.570	(8.286, 0, 8.714, 0)
3	0.364	110.478	(8.184, 0, 8.638, 0)
6	0.335	109.562	(8.099, 0, 8.574, 0)
9	0.309	108.788	(8.027, 0, 8.520, 0)
12.28	0.286	108.057	(7.959, 0, 8.469, 0)

	resources actually used		
p_0	man-weeks	material Y	material Z
0	17.000	84.144	111.998
3	16.822	83.202	110.932
6	16.673	82.415	110.037
9	16.547	81.749	109.281
12.28	16.428	81.120	108.567

Step 12. After referring to Table 3.8, the decision maker may choose a satisfying solution and then terminate the solution procedure. Otherwise, he is asked to specify a refined p_0. It is quite reasonable here to obtain a p_0 which can really satisfy the decision maker's need. Table 3.8 provides very detailed information of the optimal solution of a symmetric fuzzy linear programming problem with b_0 = 111.57 for each p_0 in the given set.

Here, let us assume that the optimal solution should be in between p_0 = 9 and 12.28. Finally, the decision maker indicates that p_0 = 10 will be perfect. Then go to Step 9.

Note.

It is necessary to compare the solutions of Problem 3 and Problem 4. The difference is shown in Table 9 and Figure 2.7.

Table 3.9. The comparison of the solutions of Problem 3 and Problem 4

Problem 3	Problem 4
θ = 0.5	0.3
Z^{**} = 114.65	108.54
x^{**} = (8.57, 0, 8.93, 0)	(8.01, 0, 8.50, 0)
resources used	
man-weeks: 17.50	16.51
material Y: 86.78	81.57
material Z: 115.01	109.03

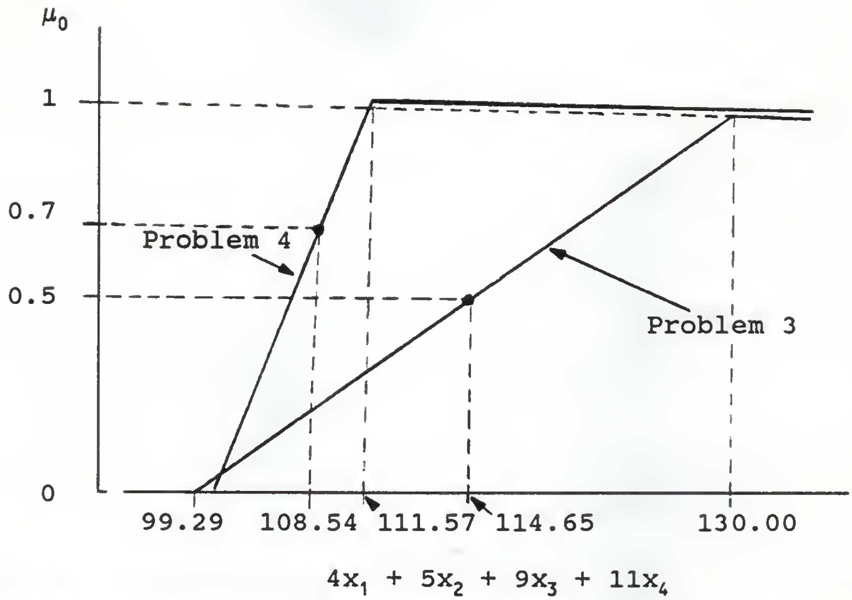


Figure 3.7. The comparison of the solutions of Problem 3 and Problem 4

CHAPTER 4 CONLUDING REMARKS AND FURTHER STUDIES

Fuzzy sets theory and fuzzy mathematical approaches have provided more reliable models to solve real-world decision making problems than did oversimplified mathematical techniques. Fuzzy mathematical approaches are not only useful to consider quantitative problems, but also to consider qualitative problems such as nature language, etc.

Among a variety of operations research approaches, linear programming is always considered as one of the most important. Therefore, in this study we investigated fuzzy linear programming as the beginning of doing research on fuzzy decision making problems.

Fuzzy linear programming problems have only been investigated for 15 years, since Tanake et al. (1974) first proposed an approach to solve a fuzzy linear programming problem with fuzzy resources. Negoita [1981] stated that there were no more then 10 papers discussing this topic. As of the present time, there are more than 200 papers existing in the literature. Because fuzzy linear programming can really provide a more flexible and more reliable analysis for the real-world problems, it will become more and more popular in the future.

It is a trend to connect operations research approaches and Artificial Intelligence to solve real-world problems. To

develop an expert LP system in order to solve a specific domain of the real-world LP problems is considered of importance. In this study, we developed an interactive fuzzy linear programming approach which is a problem-oriented and user-dependent system. Since this approach can solve a variety of LP situations, we call it an expert LP system.

The most important features of the fuzzy sets theory are operators and membership functions. But until now, we have emphasizd only the fuzzy sets with linear membership functions and using min-operator to aggregate fuzzy sets. Of course, this is just for the sake of simplicity. In some situations, the assumptions of linear membership functions and "logic and" are questionable. However, there are some other forms of membership functions which have been investigated in the literature, such as piecewise linear, exponential, hyperbolic, etc. membership functions. For operators, there are compensatory operators, min-bounded sum operator, τ-operators, "fuzzy and" operator, weighted mean operators, etc. Therefore, our further studies are to investigate various operators and membership functions which determine the flexibility and robustness of fuzzy mathematical approaches. On the other hand, in the real-world problems, multiple objectives are always concerned. As a matter of fact, an interactive fuzzy multiple objective linear programming -- an expert MLP system -- is expected. A variety of membership functions and operators will be included to satisfy various

situations and decision makers.

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INTERACTIVE FUZZY LINEAR PROGRAMMING

-- AN EXPERT LP SYSTEM

by

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ABSTRACT

Since fuzzy sets theory was proposed by Zadeh in 1965, we have been able to handle not only quantitative problems, but also qualitative problems. For qualitative examples, propositions such as "beautiful girls", "intelligent men", "creditworthy customers", etc. cannot be described well by the classical set theory. The adjectives, "beautiful", "intelligent" and "creditworthy", are vague or fuzzy. Fuzzy sets theory provides a degree-scaled function (membership function) to indicate the degree that a girl, man or customer is beautiful, intelligent or creditworthy, respectively.

Linear programming is one of the most important techniques used in decision processes involving human aspects. Therefore, in this study we have applied the fuzzy sets theory to linear programming concepts and approaches in order to improve the flexibility and robustness of traditional LP techniques. An interactive fuzzy linear programming approach investigated here is a symmetric synthesis of Zimmermann's, Werners', Verdegay's and Chanas' fuzzy linear programming approaches and additionally provides an expert LP system for solving a specific domain of real-world LP system.

Some discussions of the disadvantages of Zimmermann's and Chanas's approaches, and the comparison of Zimmermann's and Werners's solutions are provided in this studies. To overcome their disadvantages and to propose a more practical approach, a reasonable and logic step-by-step procedure should

be considered in order to reach main problems under fuzzy environments. A systematic flow chart is then developed. The decision maker therefore can more involve in his LP problem under flexible (fuzzy) resources available.

Since the fuzzy sets theory is rather human-dependent, it can be reasonably applied to OR techniques only via the participation of the decision maker. Thus an interactive concept is necessary when a LP problem is fuzzy. Besides, we have tied fuzzy LP techniques with traditional LP concepts. Finally, an interactive fuzzy linear programming approach is developed.

The interactive fuzzy linear programming system is considered as a problem-oriented and user-dependent system. It will also help the decision maker design a high-productivity system, instead of optimizing a given system.

Finally, it is worth noting that this system needs only two techniques: the simplex method and the parametric technique.